

SSUSI Auroral E-Region Algorithm

Language-Independent Description

Version 2.0

(DRAFT)

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1. Introduction

Welcome to the Language-Independent Description (LID) of the SSUSI Auroral E-Region Algorithm.

This document details the SSUSI Auroral E-Region Algorithm in a manner which is independent of any computer programming language or computer hardware architecture. It is important, though, that the reader have a basic understanding of mathematics (variables raised to a power, square-roots, power series, etc.).

1.1 The SSUSI Auroral E-Region Algorithm

The SSUSI Auroral E-Region Algorithm attempts to quantify specific characteristics of the diffuse aurora. The methods used in the quantification rely upon specific Ultra-Violet emissions as measured by the SSUSI instrument. The output products are:

- E0e The Characteristic Energy of Electrons, in units of [Kiloelectron-Volts].
- Qe The Energy Flux of Electrons, in units of [$1 \text{ erg} \cdot \text{cm}^{-2} \cdot \text{s}^{-1}$].
- E0p The Characteristic Energy of Protons, in units of [Kiloelectron-Volts].
- Qp The Energy Flux of Protons, in units of [$1 \text{ erg} \cdot \text{cm}^{-2} \cdot \text{s}^{-1}$].
- HmE The Height at which the Peak Density of Electrons occurs, in units of [Kilometers].
- NmE The Peak Density of Electrons, in units of [$\text{electrons} \cdot \text{cm}^{-3}$].
- FoE The Plasma Frequency, in units of [seconds^{-1}].

1.2 Goals of the Language-Independent Description

The driving force behind a Language-Independent Description ("LID") is the desire to preserve the intellectual knowledge which lies at the foundation of most scientific software. A LID attempts to form an agreement (or "contract" if you will) between the theorist and the implementor. The role of the theorist in creating the LID is to specify the algorithm completely, leaving no room for interpretation on the part of the implementor. The role of the implementor is to take the LID and develop it into an operational system, meeting the requirements of the particular system with respect to design methodology, maintainability, speed, and so on.

Modern programming languages do not serve as an optimal medium for LID expression. Programming languages often impose a syntax which constrains the expressive ability of the LID author. In addition, programming languages can be complex and can possess hidden subtleties. The syntax alone can place a double-requirement upon the author, because the author is forced to become both a theorist and a programmer. In the modern world, where computer programming languages and computer architectures are in a state of constant evolution, tying the expression of knowledge to a programming language can be a costly and error-prone mistake.

To summarize then, the SSUSI Auroral E-Region Algorithm Language-Independent Description attempts to:

- Clearly, and without interpretation, express the derivations of the Auroral E-Region data products.
- Provide an easily understood sequence of calculations which are broken down into "atomic" steps.
- immortalize the algorithm by transcending any particular programming language or computer architecture, thereby preserving the knowledge and portability.
- Provide a basis for testing, in that test cases can be based upon the LID, and expected results can be computed by an external method (verification from an external source).
- Enhance requirements traceability. Since each step in the LID is "atomic" and is labeled, the implementor can easily document where and how each step is accomplished. This forms a cross-reference between the code and the LID.
- Enhance maintainability. Due to the labeling of the steps in the LID, a maintainer of the code should be able to easily identify where changes should be made, and where they should not!
- Leave sufficient room for "creativity" on the part of the Implementor, so that coding the algorithm in a programming language does not become a robotic process.

1.3 Credits

This document is based entirely upon the FORTRAN implementation of the SSUSI Auroral E-Region Algorithm. The FORTRAN implementation was designed, developed, and implemented by:

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All appropriate credit should go to the aforementioned individuals, including, but not limited to, all works that their FORTRAN implementation referenced.

2. Derivations of the Auroral E-Region Data Products

This section contains the derivations of all SSUSI Auroral E-Region data products. The first subsection, "General Algorithm Expectations", exposes the linkage and preprocessing expectations common to all of the derivations. Implied subtleties are also included there.

Following the "General Algorithm Expectations" subsection are the individual data-product derivations. Each derivation begins with a listing of the input parameters required, a formal "Begin" indicator, the steps in the derivation, and a concluding "End" indicator.

Many of the steps of each derivation actually consist of a short sequence of substeps. In all cases, the substeps could actually be merged into one calculation. However, the decision to break-out the substeps was made to increase clarity and emphasize commonality amongst the steps. For example, consider the following sequence of substeps:

1.2.3.4 Calculate (StepGoal).

$$TempX = \text{Log}_e(10.0)$$

$$TempY = (TempX)^2$$

$$StepGoal = \frac{TempY}{5.0}$$

TempX and TempY are examples of "Temporary" quantities computed in substeps. The desired data product is the StepGoal at the end of each step. The "Temporary" quantities do not retain their values across step boundaries. In the above example, TempX and TempY would be undefined outside of step 1.2.3.4. Note that the StepGoal could equivalently be expressed as:

1.2.3.4 Calculate (StepGoal).

$$StepGoal = \frac{(\text{Log}_e(10.0))^2}{5.0}$$

However, as was previously mentioned, this latter style was considered to be counterproductive and tended to confuse the issue.

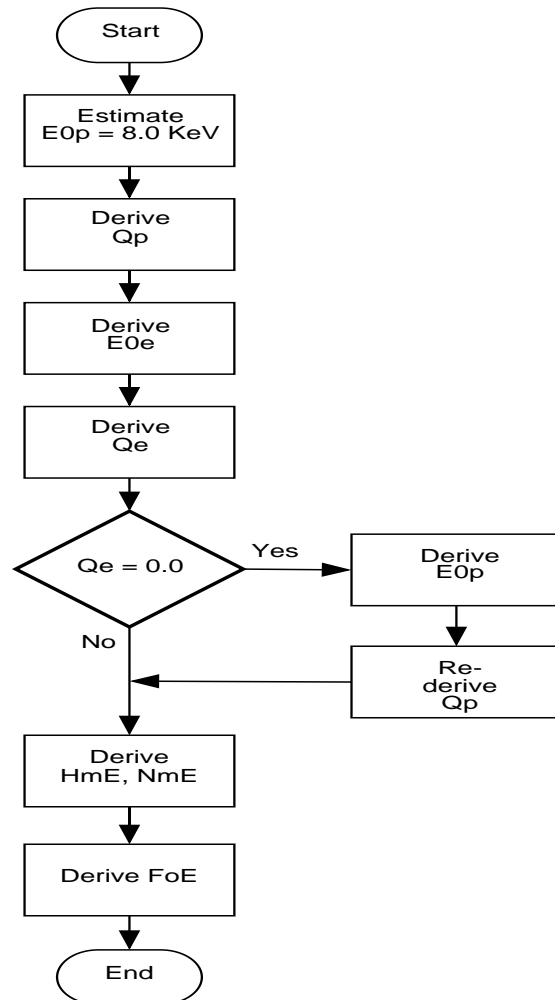
Within each step, variance calculations are provided where appropriate. They begin after the StepGoal has been calculated, and should closely match the substeps (if any) in the StepGoal calculation. Items which represent the Variance in a StepGoal are named "VStepGoal", where the "V" is a prefix appended to the StepGoal name.

2.1 General Algorithm Expectations (Required Preprocessing)

A quick inspection of the "Required Input to the Derivation" sections will reveal that there are several cross links amongst the derivations, and a seemingly-unresolvable dependency amongst the E0e and E0p data products.

The cross-linked items refer to the intermediate data items calculated in some other step (possibly in another derivation). Whatever the case, the step in which the cross-linked item is calculated can be quickly located by searching the step lists in the Table of Contents.

The dependency between the E0e and E0p data products (E0e requires E0p as input, and E0p requires E0e as input) is resolvable by performing the derivations according to the following flow-chart:



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The estimated value for E0p, 8.0 KeV as shown in the above flow-chart, can be found at the end of the E0p derivation and is labeled "for Estimation Purposes".

2.2 The Energy Flux of Protons (Qp)

2.2.1 Required Input to the Derivation

The following data-items are required, as input, by the Qp derivation:

E0p	The Characteristic Energy of Protons, in units of [Kiloelectron-Volts]. The E0p is a product of the Aurora E0p derivation.
VE0p	The Variance associated with the Characteristic Energy of Protons, in units of [Kiloelectron-Volts] ² . The VE0p is also a product of the Aurora E0p derivation.
I1216	The measured Lyman-Alpha (1216A) intensity AFTER GEOCORONAL-BACKGROUND SUBTRACTION, in units of [Rayleighs].
VI1216	The Variance associated with the measured Lyman-Alpha (1216A) intensity AFTER GEOCORONAL-BACKGROUND SUBTRACTION, in units of [Rayleighs] ² .

2.2.2 The Derivation

2.2.2.1 Begin

2.2.2.2 Calculate the Analytical Model Lyman Alpha 1216A yield due to Protons (AMLp).

The yield curve for Lyman alpha emission intensity from proton precipitation ($1 \text{ erg cm}^{-2} \text{ s}^{-1}$) was derived in the same way as the other yield curves. The model “data”, the fitted curve, and the 1σ deviations are displayed in Figure 5. As in the other case, the covariance matrix for the coefficients comes from the least squares fitting procedure. The sample standard deviations were again multiplied by two before being used to weight the mean values of Lyman alpha intensity at each characteristic energy.

$$TempX = \sum_{n=0}^3 CLYAp[n] * (E0p)^n$$

where:

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CLYAp = The Predetermined Analytical Model Coefficients of Lyman Alpha (1216A) yield versus Characteristic Energy for Protons in the Auroral Region.

E0p = The Characteristic Energy of Protons (a product of the Aurora E0p calculation).

$$AMLp = \text{Exp}(TempX)$$

where:

TempX = The Analytical Model Lyman Alpha yield (1216A; due to Protons) as a function of the Natural Log.

Exp = The Inverse Natural Log function.

Calculate the Variance in the Analytical Model Lyman Alpha yield (1216A) due to Protons (VAMLp).

$$VTempX = \sum_{j=0}^3 \sum_{i=0}^3 VLYAp[i,j] * (E0p)^{(i+j)} + VE0p * \sum_{j=0}^3 \sum_{i=0}^3 i * j * CLYAp[i] * CLYAp[j] * (E0p)^{(i+j-2)}$$

where:

VLYAp = The Predetermined Covariance Matrix corresponding to the Lyman-Alpha-Yield-Versus-Characteristic-Energy-Coefficients CLYAp(n).

CLYAp = The Predetermined Analytical Model Coefficients of Lyman-Alpha-Yield-Versus-Characteristic-Energy for Protons in the Auroral Region.

E0p = The Characteristic Energy of Protons (a product of the Aurora E0p calculation).

VE0p = The Variance associated with E0p (a product of the Aurora E0p calculation).

$$VAMLp = VTempX * (AMLp)^2$$

2.2.2.3 Calculate the Proton Energy Flux from the Proton contribution to the Lyman Alpha 1216A intensity and the Analytical Model Lyman Alpha yield (Qp).

As with LBH emissions, the Lyman alpha intensity is directly proportional to the energy flux of the incident protons. Therefore, the energy flux may be calculated by comparing the observed intensity with the yield for a 1 erg cm⁻² s⁻¹ proton aurora.

$$Qp = \frac{I1216}{AMLp}$$

where:

I1216 = The Measured Lyman Alpha (1216A) intensity (after Geocoronal-Background subtraction).

Calculate the Variance in the Proton Energy Flux (VQp).

$$VQp = \frac{VI1216 * (AMLp)^2 + VAMLp * (I1216)^2}{(AMLp)^4}$$

where:

VI1216 = The Variance in the Measured Lyman Alpha Intensity (1216A; after Geocoronal-Background subtraction).

I1216 = The Measured Lyman Alpha Intensity (1216A; after Geocoronal-Background subtraction).

2.2.2.4 End

2.3 The Characteristic Energy of Precipitating Electrons (E0e)

2.3.1 Required Input to the Derivation

The following data-items are required, as input, by the E0e derivation:

E0p	The Characteristic Energy of Protons, in units of [Kiloelectron-Volts]. The E0p is a product of the Aurora E0p derivation.
VE0p	The Variance associated with the Characteristic Energy of Protons, in units of [Kiloelectron-Volts] ² . The VE0p is also a product of the Aurora E0p derivation.
Qp	The Energy Flux of Protons, in units of [1 erg*cm-2*s-1]. The Qp is a product of the Aurora Qp derivation.
VQp	The Variance associated with the Energy Flux of Protons, in units of [1 erg*cm-2*s-1] ² . The VQp is also a product of the Aurora Qp derivation.
I1450	The measured LBH1 (1400-1500A) intensity AFTER DAYGLOW-BACKGROUND SUBTRACTION, in units of [Rayleighs].
VI1450	The Variance associated with the measured LBH1 intensity AFTER DAYGLOW-BACKGROUND SUBTRACTION, in units of [Rayleighs] ² .
I1725	The measured LBH2 (1650-1800A) intensity AFTER DAYGLOW-BACKGROUND SUBTRACTION, in units of [Rayleighs].
VI1725	The Variance associated with the measured LBH2 intensity AFTER DAYGLOW BACKGROUND SUBTRACTION, in units of [Rayleighs] ² .

2.3.2 The Derivation

2.3.2.1 Begin

2.3.2.2 Calculate the Analytical Model LBH1 Yield due to Protons (AM1p).

The LBH1 yield curve for protons was derived from calculations using a coupled proton/electron transport code [Basu et al., 1993; Strickland et al., 1993] for a variety of atmospheric conditions corresponding to a range of solar activity and geomagnetic activity levels and to seasonal variations as modeled by the MSIS-86 model thermosphere [Hedin, 1987]. In operational use, one of the principal sources of error will be the lack of information on the neutral atmosphere in

the auroral region. The transport calculations were intended to span the range of atmospheric variability and thus provide an estimate of the uncertainty due to lack of information on the auroral neutral atmosphere.

The model results for LBH1 yields along with the fitted yield curve and 1 σ deviations are shown in Figure 1. The curve was fit to the mean value of the intensities at each characteristic energy $E_{0,p}$, weighted by the reciprocal of *twice* the sample standard deviation, $1/(2\sigma_{sample})$. The factor of two is intended to allow for the unknown model uncertainties, e.g., uncertainties in cross sections and deviations from the assumed energy spectrum of the incident particles. The yield curve uncertainties were derived using the standard error propagation formulae as given by *Bevington* [1969] or *Bevington and Robinson* [1992] using the covariance matrix of the coefficients produced by the least squares fitting technique (Singular Value Decomposition as described in *Press et al.*, [1992]).

$$TempX = \sum_{n=0}^3 CLBH1p[n] * (E0p)^n$$

where:

- CLBH1p = Predetermined coefficients of LBH1 yield versus Characteristic Energy (for the Analytical Model).
- E0p = Characteristic Energy of Protons (a product of the E0p calculation in the Aurora Algorithm).

$$AM1p = \text{Exp}(TempX)$$

where:

- TempX = The Analytical Model LBH1 yield due to Protons (a function of the Natural Log).
- Exp = The Inverse Natural Log function.

Calculate the Variance in the Analytical Model LBH1 yield due to Protons (VAM1p).

$$VTempX = \sum_{j=0}^3 \sum_{i=0}^3 VLBH1p[i, j] * (E0p)^{(i+j)} +$$

$$VE0p * \sum_{j=0}^3 \sum_{i=0}^3 i * j * CLBH1p[i] * CLBH1p[j] * (E0p)^{(i+j-2)}$$

where:

VLBH1p = Predetermined Covariance Matrix corresponding to the LBH1-Yield-Versus-Characteristic Energy Coefficients CLBH1p(n).

CLBH1p = Predetermined Coefficients of LBH1 yield versus Characteristic Energy for Protons.

E0p = The Characteristic Energy of Protons (a product of the E0p calculation in the Auroral Algorithm).

VE0p = The Variance associated with E0p (a product of the E0p calculation in the Auroral Algorithm).

$$VAM1p = VTempX * (AM1p)^2$$

2.3.2.3 Calculate the Analytical Model LBH2 yield due to Protons (AM2p).

This formula was derived in the same way as the LBH1 yield described above. The model calculations and the fitted curve are also displayed in Figure 1.

$$TempX = \sum_{n=0}^3 CLBH2p[n] * (E0p)^n$$

where:

CLBH2p(n) = Predetermined coefficients of LBH2 yield versus Characteristic Energy (for the Analytical Model).

E0p = Characteristic Energy of Protons (a product of the E0p calculation in the Aurora Algorithm).

$$AM2p = \text{Exp}(TempX)$$

where:

TempX = The Analytical Model LBH2 yield due to Protons (a function of the Natural Log).

Exp = The Inverse Natural Log function.

Calculate the Variance in the Analytical Model LBH2 yield due to Protons (VAM2p).

$$VTempX = \sum_{j=0}^3 \sum_{i=0}^3 VLBH2p[i, j] * (E0p)^{(i+j)} +$$

$$VE0p * \sum_{j=0}^3 \sum_{i=0}^3 i * j * CLBH2p[i] * CLBH2p[j] * (E0p)^{(i+j-2)}$$

where:

VLBH2p = Predetermined Covariance Matrix corresponding to the LBH2-Yield-Versus-Characteristic Energy Coefficients CLBH2p(n).

CLBH2p = Predetermined Coefficients of LBH2 yield versus Characteristic Energy.

E0p = The Characteristic Energy of Protons (a product of the E0p calculation in the Auroral Algorithm).

VE0p = The Variance associated with E0p (a product of the E0p calculation in the Auroral Algorithm).

$$VAM2p = VTempX * (AM2p)^2$$

2.3.2.4 Calculate the Proton Contribution to the LBH1 intensity (PC1).

Because the production of the LBH band emission in an aurora is a linear process, the intensity is directly proportional to the energy flux of the precipitating particles. Thus, the standard yield curves described above were all derived for proton aurora with an incident energy flux, Q_p , of $1 \text{ erg cm}^{-2} \text{ s}^{-1}$. The actual LBH intensity for any given value of Q_p is obtained by multiplying the standard yield by Q_p .

$$PCI = Qp * AM1p$$

where:

Qp = The Proton Energy Flux (a product of the Qp calculation in the Aurora Algorithm).

Calculate the Variance in the Proton Contribution to the LBH1 intensity (VPC1).

$$VPC1 = VQp * (AM1p)^2 + VAM1p * (Qp)^2$$

where:

Qp = The Proton Energy Flux (a product of the Qp calculation in the Auroral Algorithm).

VQp = The Variance associated with Qp (a product of the Qp calculation in the Auroral Algorithm).

2.3.2.5 Calculate the Proton Contribution to the LBH2 intensity (PC2).

$$PC2 = Qp * AM2p$$

where:

Qp = The Proton Energy Flux (a product of the Qp calculation in the Aurora Algorithm).

Calculate the Variance in the Proton Contribution to the LBH2 intensity (VPC2).

$$VPC2 = VQp * (AM2p)^2 + VAM2p * (Qp)^2$$

where:

Qp = The Proton Energy Flux (a product of the Qp calculation in the Auroral Algorithm).

$VQp =$ The Variance associated with Qp (a product of the Qp calculation in the Auroral Algorithm).

2.3.2.6 Calculate the Electron Contribution to the LBH1 intensity (EC1).

As stated above, the production of auroral LBH band emission is a linear process, so the electron and proton contributions to the emission intensity are additive. Therefore, the electron contribution to the total intensity may be obtained by subtracting the calculated proton contribution from the measured total.

$$EC1 = I1450 - PCI$$

where:

$I1450 =$ The Measured LBH1 intensity (after Dayglow-Background subtraction).

Calculate the Variance in the Electron Contribution to the LBH1 intensity (VEC1).

$$VEC1 = VI1450 + VPCI$$

where:

$VI1450 =$ The Variance associated with the Measured LBH1 intensity (after Dayglow-Background subtraction).

2.3.2.7 Calculate the Electron Contribution to the LBH2 intensity (EC2).

$$EC2 = I1725 - PC2$$

where:

$I1725 =$ The Measured LBH2 intensity (after Dayglow-Background subtraction).

Calculate the Variance in the Electron Contribution to the LBH2 intensity (VEC2).

$$VEC2 = VI1725 + VPC2$$

where:

VI1725 = The Variance associated with the Measured LBH2 intensity (after Dayglow-Background subtraction).

2.3.2.8 Calculate the Ratio of the LBH1 and LBH2 Electron contributions (R12E).

$$R12E = \frac{EC1}{EC2}$$

Calculate the Variance in the Ratio of the LBH1 and LBH2 Electron contributions (VR12E).

$$VR12E = \frac{VEC1 * (EC2)^2 + VEC2 * (EC1)^2}{(EC2)^4}$$

2.3.2.9 Calculate the Gross Analytical Model Characteristic Energy for Electrons (GE0e).

The function relating the electron characteristic energy, $E_{0,e}$, to the ratio of the two LBH intensities was derived in much the same way as the proton LBH yield curves described above. In this case, only the electron transport part of the auroral code was used. Because the transport codes can only be run “forward,” that is, with characteristic energy as an input and auroral intensities as output, the relationship is more naturally scene as intensity as a function of characteristic energy, $I_{LBH}(E_{0,e})$. Furthermore, the sample standard deviation is most easily calculated from the scatter in intensities at a given value of $E_{0,e}$. Therefore, in order to avoid using non-linear fitting techniques, we first fit the model “data” with a function of the form

$$\frac{1}{R_{LBH}(E_{0,e})} = a_0 + a_1 E_{0,e}$$

where R_{LBH} is the ratio of the LBH1 intensity to the LBH2 intensity. Then we inverted the above function to obtain

$$E_{0,e}(R_{\text{LBH}}) = -\frac{a_0}{a_1} + \frac{1}{a_1 R_{\text{LBH}}} = c_0 + \frac{c_1}{R_{\text{LBH}}}$$

The covariance matrix for a_0 and a_1 was provided by the least squares fitting procedure, and the covariance matrix for c_0 and c_1 was derived through a generalization of the Bevington formulae:

$$\frac{\sigma^2(c_0, c_0)}{c_0^2} = \frac{\sigma^2(a_0, a_0)}{a_0^2} + \frac{\sigma^2(a_1, a_1)}{a_1^2} - 2 \frac{\sigma^2(a_0, a_1)}{a_0 a_1}$$

$$\frac{\sigma^2(c_1, c_1)}{c_1^2} = \frac{\sigma^2(a_1, a_1)}{a_1^2}$$

$$\frac{\sigma^2(c_0, c_1)}{c_0 c_1} = \frac{\sigma^2(c_1, c_0)}{c_0 c_1} = \frac{\sigma^2(a_1, a_1)}{a_1^2} - \frac{\sigma^2(a_0, a_1)}{a_0 a_1}$$

The model “data”, the fitted function, and the 1 σ deviations are displayed in Figure 2.

$$GE0e = CE0Ee[0] + \frac{CE0Ee[1]}{R12E}$$

Calculate the Variance in the Gross Analytical Model Characteristic Energy for Electrons (VGE0e).

$$VGE0e = VE0Ee[0,0] + \frac{VE0Ee[1,1]}{(CE0Ee[1])^2} + \frac{(CE0Ee[1])^2 * VR12E}{(R12E)^4} + 2 * \frac{VE0Ee[0,1]}{R12E}$$

2.3.2.10 Refine the Gross Characteristic Energy for Electrons (E0e).

When $E_{0,e}$ is very small (below 0.5 keV), the electrons deposit most of their energy in the F-layer instead of the E-layer. In addition, the energy flux associated with soft electron spectra is usually rather small. Thus, the E-layer algorithm is not really relevant below 0.5 keV and the uncertainties in $E_{0,e}$ become quite large. For these reasons we set a minimum value of 0.5 keV for $E_{0,e}$.

Because the proton contribution to the two LBH intensities is estimated using a nominal value for $E_{0,p}$, it is possible that in situations where the proton contribution dominates, the estimated proton contribution will be larger than the measured total intensity. In that case, we wish to set

the electron energy flux (see below) to zero and set the electron characteristic energy to some nominal non-zero value, which for convenience we chose to be 0.5 keV.

Whenever the value of $E_{0,e}$ is set to a nominal value, the normal error propagation formulae do not apply. In that case, we assume that the relative uncertainty of the final value of $E_{0,e}$ is 50%. The variance is the square of the absolute uncertainty, that is, $(0.5 \times E_{0,e}^{(nom)})^2 = 0.25 \times 0.5^2$.

$$E0e = \begin{cases} 0.5 & \text{for } GE0e < 0.5 \\ & \text{or } EC1 \leq 0.0 \\ & \text{or } EC2 \leq 0.0 \\ GE0e & \text{otherwise} \end{cases}$$

Calculate the Variance in the Refined Analytical Model Characteristic Energy for Electrons (VE0e).

$$VE0e = \begin{cases} \text{MAXIMUM_OF}(VGE0e, 0.25 * (0.5)^2) & \text{for } GE0e < 0.5 \\ & \text{or } EC1 \leq 0.0 \\ & \text{or } EC2 \leq 0.0 \\ VGE0e & \text{otherwise} \end{cases}$$

2.3.2.11 End

2.4 The Energy Flux of Electrons (Qe)

2.4.1 Required Input to the Derivation

The following data-items are required, as input, by the Qe derivation:

E0e	The Characteristic Energy of Electrons, in units of [Kiloelectron-Volts]. The E0e is a product of the Aurora E0e derivation.
VE0e	The Variance associated with the Characteristic Energy of Electrons, in units of [Kiloelectron-Volts] ² . The VE0e is also a product of the Aurora E0e derivation.
EC1	The Electron Contribution to the LBH1 intensity, in units of [Rayleighs]. The EC1 is an intermediate product produced during the Aurora E0e derivation.
VEC1	The Variance associated with the Electron Contribution to the LBH1 intensity, in units of [Rayleighs] ² . The VEC1 is also an intermediate product produced during the Aurora E0e derivation.
EC2	The Electron Contribution to the LBH2 intensity, in units of [Rayleighs]. The EC2 is an intermediate product produced during the Aurora E0e derivation.
VEC2	The Variance associated with the Electron Contribution to the LBH2 intensity, in units of [Rayleighs] ² . The VEC2 is also an intermediate product produced during the Aurora E0e derivation.
Qp	The Energy Flux of Protons, in units of [1 erg*cm ⁻² *s ⁻¹]. The Qp is a product of the Aurora Qp derivation.

2.4.2 The Derivation

2.4.2.1 Begin

2.4.2.2 Calculate the Analytical Model LBH1 yield due to Electrons (AM1e).

The electron yield curves were derived in the same way as the proton yield curves. The model “data” and the fitted function for LBH1 are displayed in Figure 3.

$$TempX = \sum_{n=0}^3 CLBHe[n] * (E0e)^n$$

where:

CLBHe = Predetermined Analytical Model Coefficients of LBH1 yield versus Characteristic Energy for Electrons in the Auroral Region.

E0e = The Characteristic Energy of Electrons (a product of the E0e calculation in the Auroral Algorithm).

$$AM1e = \text{Exp}(TempX)$$

where:

TempX = The Analytical Model LBH1 yield due to Electrons (a function of the Natural Log).

Exp = The Inverse Natural Log Function.

Calculate the Variance in the Analytical Model LBH1 yield due to Electrons (VAM1e).

$$VTempX = \sum_{j=0}^3 \sum_{i=0}^3 VLBHe[i,j] * (E0e)^{(i+j)} + VE0e * \sum_{j=0}^3 \sum_{i=0}^3 i * j * CLBHe[i] * CLBHe[j] * (E0e)^{(i+j-2)}$$

where:

VLBHe = Predetermined Covariance Matrix corresponding to the LBH1-Yield-Versus-Characteristic-Energy Coefficients CLBHe(n).

CLBHe = Predetermined Analytical Model Coefficients of LBH1-Yield-Versus-Characteristic-Energy for Electrons in the Auroral region.

E0e = The Characteristic Energy of Electrons (a product of the E0e calculation in the Auroral Algorithm).

$VE0e =$ The Variance associated with $E0e$ (a product of the $E0e$ calculation in the Auroral Algorithm).

$$VAM1e = VTempX * (AM1e)^2$$

2.4.2.3 Calculate the Analytical Model LBH2 yield due to Electrons (AM2e).

The electron yield curves were derived in the same way as the proton yield curves. The model “data” and the fitted function for LBH2 are displayed in Figure 3.

$$TempX = \sum_{n=0}^3 CLBH2e[n] * (E0e)^n$$

where:

$CLBH2e =$ Predetermined Analytical Model Coefficients of LBH2 yield versus Characteristic Energy for Electrons in the Auroral Region.

$E0e =$ The Characteristic Energy of Electrons (a product of the $E0e$ calculation in the Auroral Algorithm).

$$AM2e = \text{Exp}(TempX)$$

where:

$TempX =$ The Analytical Model LBH1 yield due to Electrons (a function of the Natural Log).

$\text{Exp} =$ The Inverse Natural Log Function.

Calculate the Variance in the Analytical Model LBH2 yield due to Electrons (VAM2e).

$$VTempX = \sum_{j=0}^3 \sum_{i=0}^3 VLBH2e[i,j] * (E0e)^{(i+j)} +$$

$$VE0e * \sum_{j=0}^3 \sum_{i=0}^3 i * j * CLBH2e[i] * CLBH2e[j] * (E0e)^{(i+j-2)}$$

where:

VLBH2e = Predetermined Covariance Matrix corresponding to the LBH2-Yield-Versus-Characteristic-Energy Coefficients CLBH2e(n).

CLBH2e = Predetermined Analytical Model Coefficients of LBH2-Yield-Versus-Characteristic-Energy for Electrons in the Auroral region.

E0e = The Characteristic Energy of Electrons (a product of the E0e calculation in the Auroral Algorithm).

VE0e = The Variance associated with E0e (a product of the E0e calculation in the Auroral Algorithm).

$$VAM2e = VTempX * (AM2e)^2$$

2.4.2.4 Calculate the Electron Energy Flux from the Electron Contribution to the LBH1 Intensity and the Analytical Model LBH1 Yield due to Electrons (EEF1).

Because the production of the LBH bands in aurora is a linear process, the yield is directly proportional to the energy flux of the precipitating particles. In particular, the LBH1 intensity due to electrons is just $I_{LBH1} = EEF1 \propto AM1e$. Therefore, if we set $I_{LBH1} = EC1$, the “measured” electron contribution, then we obtain the following formula.

$$EEF1 = \frac{EC1}{AM1e}$$

where:

EC1 = The Electron Contribution to the LBH1 intensity (see the Auroral E0e Derivation).

Calculate the Variance in the Electron Energy Flux for LBH1 measurements (VEEF1).

$$VEEF1 = \frac{VEC1 * (AM1e)^2 + VAM1e * (EC1)^2}{(AM1e)^4}$$

where:

VEC1 = The Variance in the Electron contribution to the LBH1 intensity (see the Auroral E0e Derivation).

EC1 = The Electron Contribution to the LBH1 intensity (see the Auroral E0e Derivation).

2.4.2.5 Calculate the Electron Energy Flux from the Electron Contribution to the LBH2 Intensity and the Analytical Model LBH2 Yield due to Electrons (EEF2).

The calculation is analogous to the calculation of EEF1, described above.

$$EEF2 = \frac{EC2}{AM2e}$$

where:

EC2 = The Electron Contribution to the LBH2 intensity (see the Auroral E0e Derivation).

Calculate the Variance in the Electron Energy Flux for LBH2 measurements (VEEF2).

$$VEEF2 = \frac{VEC2 * (AM2e)^2 + VAM2e * (EC2)^2}{(AM2e)^4}$$

where:

VEC2 = The Variance in the Electron contribution to the LBH2 intensity (see the Auroral E0e Derivation).

EC2 = The Electron Contribution to the LBH2 intensity (see the Auroral E0e Derivation).

2.4.2.6 Calculate the Gross Electron Energy Flux (GQe).

It is unlikely that EEF1 and EEF2 will be exactly the same. Therefore, we want to choose the one with the smallest relative uncertainty.

$$GQe = \begin{cases} EEF1 & \text{for } \frac{\sqrt{VEEF1}}{EEF1} \leq \frac{\sqrt{VEEF2}}{EEF2} \\ EEF2 & \text{otherwise} \end{cases}$$

Calculate the Variance in the Gross Electron Energy Flux (VGQe).

$$VGQe = \begin{cases} VEEF1 & \text{for } \frac{\sqrt{VEEF1}}{EEF1} \leq \frac{\sqrt{VEEF2}}{EEF2} \\ VEEF2 & \text{otherwise} \end{cases}$$

2.4.2.7 Refine the Gross Electron Energy Flux (Qe).

As mentioned above, when the proton contribution is much larger than the electron contribution, the calculated electron contribution, EC1 and/or EC2, may be negative. In those cases, we assume that the electron flux is negligible and we set Qe to zero. Even if EC1 and EC2 are both positive, we set Qe to zero if Qe less than 1% of the proton flux. Since equal electron and proton energy fluxes produce roughly the same LBH intensity, an electron flux this small will produce a contribution to the observed LBH intensities that is small compared to the uncertainties in the model calculations, and, therefore, it can be neglected.

$$Qe = \begin{cases} 0.0 & \text{for } EC1 \leq 0.0 \\ & \text{or } EC2 \leq 0.0 \\ & \text{or } GQe < 0.01 * Qp \\ GQe & \text{otherwise} \end{cases}$$

where:

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- EC1 = The Electron Contribution to the LBH1 Intensity (see the Auroral E0e Derivation).
- EC2 = The Electron Contribution to the LBH2 Intensity (see the Auroral E0e Derivation).
- Qp = The Proton Energy Flux (a product of the Qp calculation in the Auroral Algorithm).

Calculate the Variance in the Refined Electron Energy Flux (VQe).

$$VQe = VGQe$$

2.4.2.8 End

2.5 The Characteristic Energy of Precipitating Protons (E0p)

This derivation is performed only when Qe vanishes (when the derivation of Qe produces a zero value).

2.5.1 Required Input to the Derivation

The following data-items are required, as input, by the E0p derivation (unless the predetermined values for E0p and VE0p are desired for "Estimation Purposes"):

I1450	The measured LBH1 (1400-1500A) intensity AFTER DAYGLOW-BACKGROUND SUBTRACTION, in units of [Rayleighs].
VI1450	The Variance associated with the measured LBH1 intensity AFTER DAYGLOW-BACKGROUND SUBTRACTION, in units of [Rayleighs] ² .
I1725	The measured LBH2 (1650-1800A) intensity AFTER DAYGLOW-BACKGROUND SUBTRACTION, in units of [Rayleighs].
VI1725	The Variance associated with the measured LBH2 intensity AFTER DAYGLOW BACKGROUND SUBTRACTION, in units of [Rayleighs] ² .
CVI1450I1725	The Covariance between I1450 and I1725, in units of [Rayleighs] ² .

2.5.2 The Derivation

2.5.2.1 Begin

2.5.2.2 Calculate the Ratio of the LBH1 and LBH2 Proton contributions (R12P).

Note that R12P is the ratio of two colors (see the variance calculation for more information).

$$R12P = \frac{I1450}{I1725}$$

Calculate the Variance in the Ratio of the LBH1 and LBH2 Proton contributions (VR12P).

Since R12P is equal to the ratio of two colors, the covariance between the colors is included in the variance formula. This prevents the variance VR12P from being artificially high by subtracting-off common terms in the radiance calibration between LBH1 and LBH2. This term will be principally important from the derived on-orbit calibrations.

$$VR12P = (R12P)^2 * \left(\frac{VI1450}{(I1450)^2} + \frac{VI1725}{(I1725)^2} - 2 * \frac{CVI1450I1725}{I1450 * I1725} \right)$$

2.5.2.3 Adjust the Ratio of the LBH1 and LBH2 Proton contributions (AR12P).

Under most circumstances, the ratio R12P should be between 1 and 2. The only time it might vanish would be when I1450 is very small, so that due to statistical fluctuations a pixel might contain no counts in the LBH1 channel. In that case, the ratio will be calculated as zero, which would result in a very large value of $E_{0,p}$. To avoid this circumstance, if R12P vanishes, we set it to the smallest value we expect under most circumstances, 1. If this happens, we again assume a relative uncertainty of 50% resulting in a variance of $(0.5 \times 1.0)^2$. This scenario should be a very rare occurrence, since it can occur only when (1) the electron energy flux is negligible compared to the proton energy flux inferred from Lyman alpha, and (2) the LBH intensities are very small.

$$AR12P = \begin{cases} 1.0 & \text{for } R12P = 0.0 \\ R12P & \text{otherwise} \end{cases}$$

Calculate the Variance in the Adjusted Ratio of the LBH1 and LBH2 Proton contributions (VAR12P).

$$VAR12P = \begin{cases} \text{MAXIMUM_OF}(VR12P, 0.25 * (1.0)^2) & \text{for } R12P = 0.0 \\ VR12P & \text{otherwise} \end{cases}$$

2.5.2.4 Calculate the Gross Analytical Model Characteristic Energy for Protons (GE0p).

The relationship between $E_{0,p}$ and the ratio of LBH intensities was obtained in the same way as the electron relationship described in 2.2.2.9. The model “data”, fitted function, and 1σ deviations are displayed in Figure 4.

$$GE0p = CE0Pp[0] + \frac{CE0Pp[1]}{AR12P}$$

where:

CE0Pp = The Predetermined Coefficients of Characteristic-Energy-versus-LBH Ratio (for Protons; for the Analytical Model).

Calculate the Variance in the Gross Analytical Model Characteristic Energy for Protons (VGE0p).

$$VGE0p = VE0Pp[0,0] + \frac{VE0Pp[1,1]}{(CE0Pp[1])^2} + \frac{(CE0Pp[1])^2 * VAR12P}{(AR12P)^4} + 2 * \frac{VE0Pp[0,1]}{AR12P}$$

where:

VE0Pp = The Predetermined Covariance Matrix corresponding to the Analytical Model Coefficients-of-Characteristic-Energy-versus-LBH-Ratio (for Protons).

CE0Pp = The Predetermined Coefficients of Characteristic Energy Versus LBH Ratio (for Protons).

2.5.2.5 Refine the Gross Characteristic Energy for Protons (E0p).

When both electron and proton precipitation is present, we do not have enough information to determine all four of Qe, Qp, E0e, E0p. In that case we set E0p = 8 keV as “typical” value. When electron precipitation is negligible, we attempt to determine the actual value of E0p as described in 2.4.2.9 and 2.4.2.10 above. However, we constrain E0p to lie between 1 keV and 25 keV, since our algorithm is probably not valid outside that range, and in any case, values outside that range are rare. As in other such cases, whenever a nominal value is used instead of the calculated value, we assume a relative uncertainty of 50%, resulting in the variances listed below.

$$E0p = \begin{cases} 1.0 & \text{for } GE0p < 1.0 \\ 25.0 & \text{for } GE0p > 25.0 \\ 8.0 & \text{for Estimation Purposes} \\ GE0p & \text{otherwise} \end{cases}$$

Calculate the Variance in the Refined Characteristic Energy for Protons (VE0p).

$$VE0p = \begin{cases} \text{MAXIMUM_OF}(VGE0p, 0.25 * (1.0)^2) & \text{for } GE0p < 1.0 \\ \text{MAXIMUM_OF}(VGE0p, 0.25 * (25.0)^2) & \text{for } GE0p > 25.0 \\ 0.25 * (8.0)^2 & \text{for Estimation Purposes} \\ VGE0p & \text{otherwise} \end{cases}$$

2.5.2.6 End

2.6 The Height of the Peak Density and the Peak Density (HmE, NmE)

2.6.1 Required Input to the Derivation

The following data-items are required, as input, by the HmE and NmE derivation:

E0e	The Characteristic Energy of Electrons, in units of [Kiloelectron-Volts]. The E0e is a product of the Aurora E0e derivation.
VE0e	The Variance associated with the Characteristic Energy of Electrons, in units of [Kiloelectron-Volts] ² . The VE0e is also a product of the Aurora E0e derivation.
Qe	The Energy Flux of Electrons, in units of [1 erg*cm ⁻² *s ⁻¹]. The Qe is a product of the Aurora Qe derivation.
VQe	The Variance associated with the Energy Flux of Electrons, in units of [1 erg*cm ⁻² *s ⁻¹] ² . The VQe is also a product of the Aurora Qe derivation.
E0p	The Characteristic Energy of Protons, in units of [Kiloelectron-Volts]. The E0p is a product of the Aurora E0p derivation.
VE0p	The Variance associated with the Characteristic Energy of Protons, in units of [Kiloelectron-Volts] ² . The VE0p is also a product of the Aurora E0p derivation.
Qp	The Energy Flux of Protons, in units of [1 erg*cm ⁻² *s ⁻¹]. The Qp is a product of the Aurora Qp derivation.
VQp	The Variance associated with the Energy Flux of Protons, in units of [1 erg*cm ⁻² *s ⁻¹] ² . The VQp is also a product of the Aurora Qp derivation.
Qeuv	The Solar Activity Index, in units of [1 erg*cm ⁻² *s ⁻¹]. The Qeuv is a product of the SSUSI Daytime Algorithm (external to this document).
VQeuv	The Variance associated with the Solar Activity Index, in units of [1 erg*cm ⁻² *s ⁻¹] ² . The VQeuv is also a product of the SSUSI Daytime Algorithm (external to this document).
SZA	The Solar Zenith Angle to the current Aurora Pixel, in units of [Degrees].
VSZA	The Variance associated with the Solar Zenith Angle, in units of [Degrees] ² .

2.6.2 The Derivation

2.6.2.1 Begin

2.6.2.2 Calculate the Ratio of the Calculated Characteristic Energy and the Reference Characteristic Energy for Electrons (RCEe).

$$RCEe = \frac{E0e}{Erefe}$$

where:

E0e = The Analytical Model Characteristic Energy for Electrons (a product of the Aurora E0e calculation).

Erefe = The Predetermined Reference Characteristic Energy (for Electrons).

Calculate the Variance in the Ratio of the Calculated Characteristic Energy and the Reference Characteristic Energy for Electrons (VRCEe).

$$VRCEe = \frac{VE0e * (Erefe)^2 + VErefe * (E0e)^2}{(Erefe)^4}$$

where:

E0e = The Analytical Model Characteristic Energy for Electrons (a product of the Aurora E0e calculation).

VE0e = The Variance associated with E0e (a product of the Aurora E0e calculation).

Erefe = The Predetermined Reference Characteristic Energy (for Electrons).

VErefe = The Predetermined Variance associated with Erefe.

2.6.2.3 Calculate the Ratio of the Calculated Characteristic Energy and the Reference Characteristic Energy for Protons (RCEp).

$$RCE_p = \frac{E0_p}{Eref_p}$$

where:

$E0_p =$ The Analytical Model Characteristic Energy for Protons (a product of the Aurora $E0_p$ calculation).

$Eref_p =$ The Predetermined Reference Characteristic Energy (for Protons).

Calculate the Variance in the Ratio of the Calculated Characteristic Energy and the Reference Characteristic Energy for Protons (VRCE_p).

$$VRCE_p = \frac{VE0_p * (Eref_p)^2 + VEref_p * (E0_p)^2}{(Eref_p)^4}$$

where:

$E0_p =$ The Analytical Model Characteristic Energy for Protons (a product of the Aurora $E0_p$ calculation).

$VE0_p =$ The Variance associated with $E0_p$ (a product of the Aurora $E0_p$ calculation).

$Eref_p =$ The Predetermined Reference Characteristic Energy (for Protons).

$VEref_p =$ The Predetermined Variance associated with $Eref_p$.

2.6.2.4 Calculate the Log-Base-10 of the Ratio of the Calculated Characteristic Energy for Electrons (LRCE_e).

$$LRCE_e = \frac{\text{Log}_e(RCE_e)}{\text{Log}_e(10)}$$

where:

$\text{Log}_e =$ The Natural Log function.

Calculate the Variance in the Log-Base-10 of the Ratio of the Calculated Characteristic Energy for Electrons (VLRCEe).

$$VLRCEe = \frac{VRCEe}{(RCEe * \text{Log}_e(10))^2}$$

where:

$\text{Log}_e =$ The Natural Log function.

2.6.2.5 Calculate the Log-Base-10 of the Ratio of the Calculated Characteristic Energy for Protons (LRCEp).

$$LRCEp = \frac{\text{Log}_e(RCEp)}{\text{Log}_e(10)}$$

where:

$\text{Log}_e =$ The Natural Log function.

Calculate the Variance in the Log-Base-10 of the Ratio of the Calculated Characteristic Energy for Protons (VLRCEp).

$$VLRCEp = \frac{VRCEp}{(RCEp * \text{Log}_e(10))^2}$$

where:

$\text{Log}_e =$ The Natural Log function.

2.6.2.6 Calculate the Peak Auroral Ionization Production Rate Height for Electrons (PPRHe).

The auroral E-region chemistry code that is part of the auroral particle transport package [Strickland *et al.*, 1993] described above, was run for a number of auroral situations. We found that the log of the height of the peak production was well represented by a linear function of

characteristic energy. Figure 6 shows the model “data” and the fitted function. As in all other cases, the sample standard deviation was doubled before using it to weight the data in the least squares fitting procedure. This allows for model uncertainties due to such things as uncertainties in reaction rates, etc.

$$TempX = \sum_{n=0}^1 CHMAXe[n] * (LRCEe)^n$$

where:

CHMAXe = The Predetermined Analytical Model Coefficients of Peak-Auroral-Ionization-Production-Rate-Height-Versus-Characteristic-Energy (for Electrons).

$$PPRHe = (10)^{TempX}$$

where:

TempX = The Peak Auroral Ionization Production Rate Height (as a function of Log10).

Calculate the Variance in the Peak Auroral Ionization Production Rate Height for Electrons (VPPRHe).

$$VTempX = \sum_{j=0}^1 \sum_{i=0}^1 VHMAXe[i,j] * (LRCEe)^{(i+j)} + \\
 VLRCEe * \sum_{j=0}^1 \sum_{i=0}^1 i * j * CHMAXe[i] * CHMAXe[j] * (LRCEe)^{(i+j-2)}$$

where:

VHMAXe = The Predetermined Covariance Matrix corresponding to the Peak-Auroral-Ionization-Production-Rate-Height-Versus-Characteristic-Energy Coefficients CHMAXe(n).

CHMAXe = The Predetermined Analytical Model Coefficients of Peak Auroral Ionization Production Rate Height versus Characteristic Energy (for Electrons).

$$VPPRHe = \left((10)^{TempX} * \text{Log}_e(10) \right)^2 * VTempX$$

where:

$\text{Log}_e =$ The Natural Log function.

2.6.2.7 Calculate the Peak Auroral Ionization Production Rate Height for Protons (PPRHp).

This relation was derived in the same way as the electron relation (2.6.2.6). Figure 7 shows the model “data” and the fitted function.

$$TempX = \sum_{n=0}^1 CHMAXp[n] * (LRCEp)^n$$

where:

CHMAXp = The Predetermined Analytical Model Coefficients of Peak-Auroral-Ionization-Production-Rate-Height-Versus-Characteristic-Energy (for Protons).

$$PPRHp = (10)^{TempX}$$

where:

TempX = The Peak Auroral Ionization Production Rate Height (as a function of Log10).

Calculate the Variance in the Peak Auroral Ionization Production Rate Height for Protons (VPPRHp).

$$VTempX = \sum_{j=0}^1 \sum_{i=0}^1 VHMAXp[i,j] * (LRCEp)^{(i+j)} + VLRCEp * \sum_{j=0}^1 \sum_{i=0}^1 i * j * CHMAXp[i] * CHMAXp[j] * (LRCEp)^{(i+j-2)}$$

where:

VHMAXp = The Predetermined Covariance Matrix corresponding to the Peak-Auroral-Ionization-Production-Rate-Height-versus-Characteristic-Energy Coefficients CHMAXp(n).

CHMAXp = The Predetermined Analytical Model Coefficients of Peak Auroral Ionization Production Rate Height versus Characteristic Energy (for Protons).

$$VPPRH_p = \left((10)^{TempX} * \text{Log}_e(10) \right)^2 * VTempX$$

where:

Log_e = The Natural Log function.

2.6.2.8 Calculate the Peak Auroral Ionization Production Rate for Electrons, for an Electron Energy Flux of 1 erg*cm-2*s-1 (PPR1e).

The same model runs that generated the “data” used for the height of the production peak also generated “data” for this function. The model “data” and the fitted function are displayed in Figure 8. Because the production rate is directly proportional to the incident particle energy flux, we only need to model the relationship for a flux of 1 erg cm⁻² s⁻¹. The production rate for any other flux can be obtained by multiplying this function by the actual energy flux.

$$TempX = \sum_{n=0}^2 CPMAX_e[n] * (LRCE_e)^n$$

where:

CPMAX_e = The Predetermined Analytical Model Coefficients of Peak Auroral Ionization Production Rate Versus Characteristic Energy (for Electrons in the Aurora).

$$TempY = (10)^{TempX}$$

$$PPR1e = TempY * PREF_e$$

where:

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PREFe = The Predetermined Reference Auroral Ionization Production Rate for the Analytical Model of Peak Auroral Ionization Production Rate (due to Electrons).

Calculate the Variance in the Peak Auroral Ionization Production Rate for Electrons, for an Electron Energy Flux of 1 erg*cm-2*s-1 (VPPRIe).

$$VTempX = \sum_{j=0}^2 \sum_{i=0}^2 VPMAXe[i, j] * (LRCEe)^{(i+j)} + VLRCEe * \sum_{j=0}^2 \sum_{i=0}^2 i * j * CPMAXe[i] * CPMAXe[j] * (LRCEe)^{(i+j-2)}$$

where:

VPMAXe = The Predetermined Covariance Matrix corresponding to the Peak-Auroral-Ionization-Production-Rate-versus-Characteristic-Energy Coefficients CPMAXe(n).

CPMAXe = The Predetermined Analytical Model Coefficients of Peak-Auroral-Ionization-Production-Rate-versus-Characteristic-Energy (for Electrons).

$$VTempY = \left((10)^{TempX} * \text{Log}_e(10) \right)^2 * VTempX$$

where:

Log_e = The Natural Log function.

$$VPPRIe = VTempY * (PREFe)^2 + VPREFe * (TempY)^2$$

where:

PREFe = The Predetermined Reference Auroral Ionization Production Rate for the Analytical Model of Peak Auroral Ionization Production Rate (due to Electrons).

VPREFe = The Predetermined Variance associated with PREFe.

2.6.2.9 Calculate the Peak Auroral Ionization Production Rate for Protons, for a Proton Energy Flux of 1 erg*cm-2*s-1 (PPR1p).

This function was derived in the same way as the electron relationship (2.6.2.8). Figure 9 shows the model “data” and the fitted function.

$$TempX = \sum_{n=0}^2 CPMAXp[n] * (LRCEp)^n$$

where:

CPMAXp = The Predetermined Analytical Model Coefficients of Peak Auroral Ionization Production Rate Versus Characteristic Energy (for Protons in the Aurora).

$$TempY = (10)^{TempX}$$

$$PPR1p = TempY * PREFp$$

where:

PREFp = The Predetermined Reference Auroral Ionization Production Rate for the Analytical Model of Peak Auroral Ionization Production Rate (due to Protons).

Calculate the Variance in the Peak Auroral Ionization Production Rate for Protons, for a Proton Energy Flux of 1 erg*cm-2*s-1 (VPPR1p).

$$VTempX = \sum_{j=0}^2 \sum_{i=0}^2 VPMAXp[i, j] * (LRCEp)^{(i+j)} + \\
 VLRCEp * \sum_{j=0}^2 \sum_{i=0}^2 i * j * CPMAXp[i] * CPMAXp[j] * (LRCEp)^{(i+j-2)}$$

where:

VPMAXp = The Predetermined Covariance Matrix corresponding to the Peak-Auroral-Ionization-Production-Rate-versus-Characteristic-Energy Coefficients CPMAXp(n).

CPMAX_p = The Predetermined Analytical Model Coefficients of Peak-Auroral-Ionization-Production-Rate-versus-Characteristic-Energy (for Protons).

$$VTempY = \left((10)^{TempX} * \text{Log}_e(10) \right)^2 * VTempX$$

where:

Log_e = The Natural Log function.

$$VPPR1p = VTempY * (PREFp)^2 + VPREFp * (TempY)^2$$

where:

PREFp = The Predetermined Reference Auroral Ionization Production Rate for the Analytical Model of Peak Auroral Ionization Production Rate (due to Protons).

VPREFp = The Predetermined Variance associated with PREFp.

2.6.2.10 Calculate the Scale Height of Auroral Ionization Production Rate due to Electrons (SHPre).

The same model runs that generated the “data” used for production peak and peak height described above were used to investigate the *shape* of the altitude profile of the production rate. Since we are using a Chapman function to describe the shape of the production profile, the parameter that controls the shape is the scale height. In fact, the height integrated (or column) ionization rate is independent of the characteristic energy, but the model calculations show that the peak ionization rate is not. It can be shown [Daniell and Brown, 1994] that the height integrated ionization (assuming a Chapman function) is proportional to the product of the peak ionization rate and the scale height. Therefore, the scale height is proportional to the ratio of the integrated ionization rate to the peak ionization rate.

$$SHPre = \frac{SHPF * SHPCe}{PPR1e}$$

where:

SHPF = The Predetermined Scale Height Proportionality Factor (independent of Electron or Proton particle type).

SHPCe = The Predetermined Proportionality Constant for the Calculation of the Scale Height of the Auroral Ionization Production Rate (due to Electrons).

Calculate the Variance in the Scale Height of the Auroral Ionization Production Rate due to Electrons (VSHPre).

$$VSHPre = \frac{(SHPF)^2 * (VSHPCe * (PPRe)^2 + VPPRe * (SHPCe)^2)}{(PPRe)^4}$$

where:

SHPF = The Predetermined Scale Height Proportionality Factor (independent of Electron or Proton particle type).

SHPCe = The Predetermined Proportionality Constant for the Calculation of the Scale Height of the Auroral Ionization Production Rate (due to Electrons).

VSHPCe = The Predetermined Variance associated with SHPCe.

2.6.2.11 Calculate the Scale Height of Auroral Ionization Production Rate due to Protons (SHPRp).

The same arguments used to derive the expression for the scale height of the electron produced production rate (2.6.2.10) apply to the proton produced production rate.

$$SHPRp = \frac{SHPF * SHPCp}{PPRp}$$

where:

SHPF = The Predetermined Scale Height Proportionality Factor (independent of Electron or Proton particle type).

SHPCp = The Predetermined Proportionality Constant for the Calculation of the Scale Height of the Auroral Ionization Production Rate (due to Protons).

Calculate the Variance in the Scale Height of the Auroral Ionization Production Rate due to Protons (*VSHPR_p*).

$$VSHPR_p = \frac{(SHPF)^2 * (VSHPC_p * (PPR1_p)^2 + VPPR1_p * (SHPC_p)^2)}{(PPR1_p)^4}$$

where:

SHPF = The Predetermined Scale Height Proportionality Factor (independent of Electron or Proton particle type).

SHPC_p = The Predetermined Proportionality Constant for the Calculation of the Scale Height of the Auroral Ionization Production Rate (due to Protons).

VSHPC_p = The Predetermined Variance associated with SHPC_p.

2.6.2.12 Calculate the Electron Peak Auroral Ionization Production Rate for an Electron Energy Flux *Q_e* (*PPRQ_e*).

Since the ionization rate is proportional to the incident energy flux, the actual production rate is obtained simply by multiplying the production rate for 1 erg cm⁻² s⁻¹ by the actual energy flux.

$$PPRQ_e = Q_e * PPR1_e$$

where:

Q_e = The Analytical Model Electron Energy Flux (a product of the Auroral *Q_e* calculation).

Calculate the Variance in the Electron Peak Auroral Ionization Production Rate for an Electron Energy Flux *Q_e* (*VPPRQ_e*).

$$VPPRQ_e = VQ_e * (PPR1_e)^2 + VPPR1_e * (Q_e)^2$$

where:

$Q_e =$ The Analytical Model Electron Energy Flux (a product of the Aurora Q_e calculation).

$VQ_e =$ The Variance associated with Q_e (a product of the Aurora Q_e calculation).

2.6.2.13 Calculate the Proton Peak Auroral Ionization Production Rate for a Proton Energy Flux Q_p (PPR Q_p).

The expression and its justification are the same as for electrons (2.6.2.12).

$$PPRQ_p = Q_p * PPR1_p$$

where:

$Q_p =$ The Analytical Model Proton Energy Flux (a product of the Auroral Q_p calculation).

Calculate the Variance in the Proton Peak Auroral Ionization Production Rate for a Proton Energy Flux Q_p (VPPR Q_p).

$$VPPRQ_p = VQ_p * (PPR1_p)^2 + VPPR1_p * (Q_p)^2$$

where:

$Q_p =$ The Analytical Model Proton Energy Flux (a product of the Aurora Q_p calculation).

$VQ_p =$ The Variance associated with Q_p (a product of the Aurora Q_p calculation).

2.6.2.14 Calculate the Solar-Photon Peak Photoionization Production Rate (PPPR h).

The peak photoionization rate is proportional to the solar EUV flux.

$$PPPRh = Q_{euv} * PPPR_{subsolar}$$

where:

Q_{euv} = The Solar Activity Index (a product of the SSUSI Daytime Algorithm).

$PPPR_{subsolar}$ = The Predetermined Peak Photoionization Production Rate (due to Solar Photons) at the Subsolar Point.

Calculate the Variance in the Solar-Photon Peak Photoionization Production Rate (VPPPRh).

$$VPPPRh = VQ_{euv} * (PPPR_{subsolar})^2 + VPPPR_{subsolar} * (Q_{euv})^2$$

where:

Q_{euv} = The Solar Activity Index (a product of the SSUSI Daytime Algorithm).

VQ_{euv} = The Variance associated with Q_{euv} (a product of the SSUSI Daytime Algorithm).

$PPPR_{subsolar}$ = The Predetermined Peak Photoionization Production Rate (due to Solar Photons) at the Subsolar Point.

$VPPPR_{subsolar}$ = The Predetermined Variance associated with $PPPR_{subsolar}$.

2.6.2.15 Calculate an Electron Density Profile from 90 km to 150 km, at 5 km increments. Repeat all of the following substeps for every element in the profile.

See Figure 10 for a representation of the geometry involved in calculating the photoionization rate. The production rate altitude profiles are assumed to be Chapman functions. For electrons and protons, the production rate has the form

$$PR(z) = PPRQ * \text{Exp}[1 - RHPR - \text{Exp}(-RHPR)]$$

$$RHPR = \frac{EA - PPRH}{SHPR}$$

For photoionization, it takes the form

$$PR(z) = PPRQ * \text{Exp}[1 - RHPR - GIF * \text{Exp}(-RHPR)]$$

where *GIF* is the Chapman Grazing Incidence Function (see Appendix). Since the ionization process is linear, the production rates calculated for each source (electrons, protons, photons) may be added together to obtain the total production rate (*PRtotal*).

Electron density is obtained using an effective recombination coefficient (*RC*, see 2.6.2.15.9) so that the electron density at altitude *z* is obtained from

$$ED(z) = \sqrt{\frac{PRtotal(z)}{RC(z)}}$$

2.6.2.15.1 Calculate the Ratio of the Radial Distance to the Neutral Scale Height (ROSH).

$$TempX = MRE + EA$$

where:

- TempX = The Total Radius from the Center of the Earth to the Observer.
- MRE = The Predetermined Mean Radius of the Earth.
- EA = The Altitude of the current element in the Electron Density Profile.

$$ROSH = \frac{TempX}{HN}$$

where:

- HN = The Predetermined Neutral Scale Height.

Calculate the Variance in the Ratio of the Radial Distance to the Neutral Scale Height (VROSH).

$$VTempX = VMRE + VEA$$

where:

V_{TempX} = The Variance associated with the Total Radius from the Center of the Earth to the Observer.

V_{MRE} = The Predetermined Variance associated with the Mean Radius of the Earth.

V_{EA} = The Predetermined Variance associated with the Altitude of the Current Element in the Electron Density Profile.

$$V_{ROSH} = \frac{V_{TempX} * (HN)^2 + V_{HN} * (TempX)^2}{(HN)^4}$$

where:

HN = The Predetermined Neutral Scale Height.

V_{HN} = The Predetermined Variance in the Neutral Scale Height.

2.6.2.15.2 Calculate the Reduced Height for the Auroral Ionization Production Rate due to Electrons (RHPR_e).

$$TempX = EA - PPRHe$$

where:

EA = The Altitude of the Current Element in the Electron Density Profile.

$$RHPR_e = \frac{TempX}{SHPR_e}$$

Calculate the Variance in the Reduced Height for the Auroral Ionization Production Rate due to Electrons (VRHPR_e).

$$V_{TempX} = V_{EA} + V_{PPRHe}$$

where:

VEA = The Predetermined Variance associated with the Altitude of the Current Element in the Electron Density Profile.

$$VRHPR_e = \frac{VTempX * (SHPR_e)^2 + VSHPR_e * (TempX)^2}{(SHPR_e)^4}$$

2.6.2.15.3 Calculate the Reduced Height for the Auroral Ionization Production Rate due to Protons (RHPRp).

$$TempX = EA - PPRHp$$

where:

EA = The Altitude of the Current Element in the Electron Density Profile.

$$RHPR_p = \frac{TempX}{SHPR_p}$$

Calculate the Variance in the Reduced Height for the Auroral Ionization Production Rate due to Protons (VRHPRp).

$$VTempX = VEA + VPPRHp$$

where:

VEA = The Predetermined Variance associated with the Altitude of the Current Element in the Electron Density Profile.

$$VRHPR_p = \frac{VTempX * (SHPR_p)^2 + VSHPR_p * (TempX)^2}{(SHPR_p)^4}$$

2.6.2.15.4 Calculate the Reduced Height for the Photoionization Production Rate due to Solar Photons (RHPRh).

$$TempX = EA - HO$$

where:

EA = The Altitude of the Current Element in the Electron Density Profile.

HO = The Predetermined Height of the Peak Photoionization Production Rate (due to Solar Photons) at the Subsolar Point.

$$RHPRh = \frac{TempX}{HN}$$

where:

HN = The Predetermined Neutral Scale Height.

Calculate the Variance in the Reduced Height for the Photoionization Production Rate due to Solar-Photons (VRHPRh).

$$VTempX = VEA + VHO$$

where:

VEA = The Predetermined Variance associated with the Altitude of the Current Element in the Electron Density Profile.

VHO = The Predetermined Variance associated with the Height of the Peak Photoionization Production Rate (due to Solar Photons) at the Subsolar Point.

$$VRHPRh = \frac{VTempX * (HN)^2 + VHN * (TempX)^2}{(HN)^4}$$

where:

HN = The Predetermined Neutral Scale Height.

VHN = The Predetermined Variance associated with HN.

2.6.2.15.5 Calculate the Auroral Ionization Production Rate due to Electrons (PRe).

$$TempX = 1 - RHPRe - \text{Exp}(-RHPRe)$$

where:

RHPRe = The Reduced Height for the Auroral Ionization Production Rate due to Electrons.

Exp = The Inverse Natural Log function.

$$PRe = PPRQe * \text{Exp}(TempX)$$

where:

PPRQe = The Electron Peak Auroral Ionization Production Rate for an Electron Energy Flux Qe.

Exp = The Inverse Natural Log function.

Calculate the Variance in the Auroral Ionization Production Rate due to Electrons (VPre).

$$VPre = \text{Exp}(2 * TempX) * \left(VPPRQe + VRHPRe * \left(PPRQe * (\text{Exp}(-RHPRe) - 1) \right)^2 \right)$$

where:

PPRQe = The Electron Peak Auroral Ionization Production Rate for an Electron Energy Flux Qe.

VPPRQe = The Variance associated with PPRQe.

RHPRe = The Reduced Height for the Auroral Ionization Production Rate due to Electrons.

VRHPRe = The Variance associated with RHPRe.

Exp = The Inverse Natural Log function.

2.6.2.15.6 Calculate the Auroral Ionization Production Rate due to Protons (PRp).

$$TempX = 1 - RHPRp - \text{Exp}(-RHPRp)$$

where:

RHPRp = The Reduced Height for the Auroral Ionization Production Rate due to Protons.

Exp = The Inverse Natural Log function.

$$PRp = PPRQp * \text{Exp}(TempX)$$

where:

PPRQp = The Proton Peak Auroral Ionization Production Rate for a Proton Energy Flux Qp.

Exp = The Inverse Natural Log function.

Calculate the Variance in the Auroral Ionization Production Rate due to Protons (VPRp).

$$VPRp = \text{Exp}(2 * TempX) * \left(VPPRQp + VRHPRp * \left(PPRQp * \left(\text{Exp}(-RHPRp) - 1 \right) \right)^2 \right)$$

where:

PPRQp = The Proton Peak Auroral Ionization Production Rate for a Proton Energy Flux Qp.

VPPRQp = The Variance associated with PPRQp.

- RHPR_p = The Reduced Height for the Auroral Ionization Production Rate due to Protons.
- VRHPR_p = The Variance associated with RHPR_p.
- Exp = The Inverse Natural Log function.

2.6.2.15.7 Calculate the Photoionization Production Rate due to Solar Photons (PRh).

$$TempX = 1 - RHPRh - GIF * Exp(-RHPRh)$$

where:

- RHPRh = The Reduced Height for the Photoionization Production Rate due to Solar Photons.
- Exp = The Inverse Natural Log function.
- GIF = The Value calculated by the Chapman Grazing Incidence Function derivation (see the Appendix). The following values should be used as input into the GIF derivation:

For GIF Radius $\xrightarrow{Use} ROSH$

For GIF VRadius $\xrightarrow{Use} VROSH$

For GIF SZA $\xrightarrow{Use} SZA$

For GIF VSZA $\xrightarrow{Use} VSZA$

$$PRh = PPPRh * Exp(TempX)$$

where:

- PPPRh = The Solar-Photon Peak Photoionization Production Rate.
- Exp = The Inverse Natural Log function.

Calculate the Variance in the Photoionization Production Rate due to Solar Photons (VPRh).

$$VPRh = VPPPRh * \text{Exp}(2 * \text{Temp}X) + (PRh)^2 * \left(\frac{VRHPRh * (1 - \text{GIF} * \text{Exp}(-RHPRh))^2 + VGIF * \text{Exp}(-2 * RHPRh)}{\text{Exp}(-2 * RHPRh)} \right)$$

where:

- VPPPRh = The Variance associated with the Solar-Photon Peak Photoionization Production Rate.
- Exp = The Inverse Natural Log function.
- RHPRh = The Reduced Height for the Photoionization Production Rate due to Solar Photons.
- VRHPRh = The Variance associated with RHPRh.
- GIF = The Value calculated by the Chapman Grazing Incidence Function derivation (see the Appendix). The following values should be used as input into the GIF derivation:

For GIF Radius $\xrightarrow{\text{Use}}$ *ROSH*

For GIF VRadius $\xrightarrow{\text{Use}}$ *VROSH*

For GIF SZA $\xrightarrow{\text{Use}}$ *SZA*

For GIF VSZA $\xrightarrow{\text{Use}}$ *VSZA*

- VGIF = The Variance associated with the GIF value calculated by the Chapman Grazing Incidence Function derivation (see the Appendix).

2.6.2.15.8 Calculate the Total Ionization Production Rate (PRtotal).

$$PRtotal = PRe + PRp + PRh$$

Calculate the Variance in the Total Ionization Production Rate (VPRtotal).

$$VPRtotal = VPRE + VPRp + VPRh$$

2.6.2.15.9 Calculate the Effective Recombination Coefficient (RC).

The auroral E-layer profile is highly variable because it can be the product of three different ionization sources (electrons, protons, and photons.) Consequently, the shape of the electron density profile (as opposed to the ionization rate profile shapes) is not accurately represented by a simple function such as a Chapman function. Therefore, it is best to calculate the electron density from a chemical model, rather than to attempt to store E-layer parameters for a complete range of electron, proton, and photoionization parameters. On the other hand, E-layer chemistry is not simple either, so a complete chemical description is also out of the question. Fortunately, it turns out that the concept of an effective recombination rate is applicable.

Figure 11 shows the effective recombination rate as a function of altitude for a variety of model runs. The effective recombination rate, RC , at any altitude z was obtained by dividing the production rate at that altitude by the square of the electron density calculated from a full chemical model.

$$RC(z) = \frac{PR_{total}(z)}{ED(z)}$$

Remarkably, the rate is nearly independent of precipitating particle characteristics above an altitude of about 108 km. Below that altitude, the precise shape depends on the hardness of the spectrum and other factors. However, the shape, though complex, tends to oscillate about an average value. This is not true for the softest spectra, but for these the electron density peaks above 108 km, where the effective recombination coefficient is independent of the ionization source. Therefore, we have adopted a “universal” functional form for the recombination coefficient: an exponential decrease with altitude above 108 km and a constant below 103 km. While this form does *not* accurately reproduce the complete electron density profile, it does reproduce the profile *near the density peak*, which is all that is necessary since the algorithm is required to produce only the peak density and the altitude of the peak.

$$RC = \begin{cases} PERC & \text{for } EA \leq PERCA \\ PERC * \exp\left(\frac{-(EA - PERCA)}{SHRC}\right) & \text{otherwise} \end{cases}$$

where:

PERC = The Predetermined Peak Effective Recombination Coefficient.

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- PERCA = The Predetermined Altitude of the Peak Effective Recombination Coefficient.
- EA = The Altitude of the Current Element in the Electron Density Profile.
- SHRC = The Predetermined Scale Height of the Effective Recombination Coefficient.
- Exp = The Inverse Natural Log function.

Calculate the Variance in the Effective Recombination Coefficient (VRC).

$$VRC = (RC)^2 * \left(\left(\frac{VEA + VPERCA + VSHRC * (RCEXP)^2}{(SHRC)^2} \right) + \left(\frac{VPERC}{(PERC)^2} \right) \right)$$

where:

- VEA = The Predetermined Variance associated with the Altitude of the Current Element in the Electron Density Profile.
- VPERCA = The Predetermined Variance associated with the Altitude of the Peak Effective Recombination Coefficient.
- VSHRC = The Predetermined Variance associated with the Scale Height of the Effective Recombination Coefficient.
- VPERC = The Predetermined Variance associated with the Peak Effective Recombination Coefficient.
- PERC = The Predetermined Peak Effective Recombination Coefficient.
- SHRC = The Predetermined Scale Height of the Effective Recombination Coefficient.

$$RCEXP = \begin{cases} 0 & \text{for } EA \leq PERCA \\ \frac{EA - PERCA}{SHRC} & \text{otherwise} \end{cases}$$

where:

EA = The Altitude of the Current Element in the Electron Density Profile.

PERCA = The Predetermined Altitude of the Peak Effective Recombination Coefficient.

RCEXP = The Effective Recombination Coefficient Exponent.

2.6.2.15.10 Calculate the Electron Density for the current element in the Profile (ED).

$$TempX = \frac{PRtotal}{RC}$$

$$TempY = \begin{cases} 1.0 & \text{for } TempX < 1.0 \\ TempX & \text{otherwise} \end{cases}$$

$$ED = \sqrt{TempY}$$

where:

ED = The Electron Density corresponding to a given altitude in the Profile.

Calculate the Variance in the Electron Density for the current element in the Profile (VED).

$$VTempX = \frac{VPRtotal * (RC)^2 + VRC * (PRtotal)^2}{(RC)^4}$$

$$VTempY = \begin{cases} \text{MAXIMUM_OF}(VTempX, 0.25 * (1.0)^2) & \text{for } TempX < 1.0 \\ VTempX & \text{otherwise} \end{cases}$$

$$VED = \frac{VTempY}{4.0 * TempY}$$

2.6.2.16 Search over the Previously-Established Profile for the Maximum Internal Peak Electron Density and the Height at which the Peak Occurs (HmE, NmE).

As stated above, the E-layer electron density profile is highly variable, and there are likely to be many pixels for which there is no well defined peak electron density. Under those circumstances, the algorithm returns a nominal value of 110 km for HmE and sets NmE to the electron density at 110 km.

$$HmE = \begin{cases} 110.0 & \text{for } No\ Peak\ Found \\ EDAP & \text{otherwise} \end{cases}$$

where:

EDAP = The Altitude corresponding to the Peak Electron Density (the Peak found by searching the Electron Density Profile).

$$NmE = ED[HmE]$$

where:

ED = The Electron Density Profile.

Calculate the Variance in the Maximum Internal Peak Electron Density and the Height at which the Peak Occurs (VHmE, VNmE).

$$VHmE = \frac{(DEA)^2}{2.0}$$

where:

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DEA = The Predetermined Delta-Change increment of the
Altitudes corresponding to Elements in the current Electron
Density Profile.

$$VNmE = VED[HmE]$$

where:

VED = The Variances associated with the members of the Electron
Density Profile.

2.6.2.17 End

2.7 The Plasma Frequency (FoE)

2.7.1 Required Input to the Derivation

The following data-items are required, as input, by the FoE derivation:

NmE	The Peak Density, in units of [centimeters-3]. NmE is a product of the Aurora NmE derivation.
VNmE	The Variance associated with the Peak Density, in units of [centimeters-3] ² . VNmE is a product of the Aurora NmE derivation.

2.7.2 The Derivation

2.7.2.1 Begin

2.7.2.2 Calculate the Plasma Frequency (FoE).

$$FoE = (8.98 * (10)^3) * \sqrt{NmE}$$

Calculate the Variance in the Plasma Frequency (VFoE).

$$VFoE = \frac{(2.01601 * (10)^7) * VNmE}{NmE}$$

2.7.2.3 End

3. Appendix

3.1 The Chapman Grazing Incidence Function (GIF)

The treatment of the Chapman Grazing Incidence Function that guided the development of this algorithm may be found in *Rishbeth and Garriott* [1969]. Refer to Figure 10 for the geometry involved in this calculation. The error function approximation (*TempD*) is from *Abramowitz and Stegun* [1964].

3.1.1 Required Input to the Derivation

The following data-items are required, as input, by the GIF derivation:

Radius	The Ratio of the Radial Distance to the Neutral Density Scale Height [unitless].
VRadius	The Variance associated with the Ratio of the Radial Distance to the Neutral Density Scale Height [unitless].
SZA	The Solar Zenith Angle at the Observer, in units of [Degrees].
VSZA	The Variance associated with the Solar Zenith Angle at the Observer, in units of [Degrees] ² .

3.1.2 The Derivation

3.1.2.1 Begin

3.1.2.2 Calculate the Grazing Incidence Function (GIF).

$$TempA = \frac{Radius * (\text{Cos}(SZA))^2}{2}$$

$$TempB = \frac{Radius * Pi * \text{Sin}(SZA)}{2}$$

where:

$P_i =$ Predetermined value of P_i .

$$TempC = \frac{1}{1 + Cterm * \sqrt{TempA}}$$

where:

$Cterm =$ Predetermined scalar applied to the argument of the GIF error-function power-series approximation.

$$TempD = \sum_{n=0}^4 Cgif[n] * (TempC)^{(n+1)}$$

where:

$Cgif =$ Predetermined coefficients of the GIF error-function power-series approximation.

$$GIF = \begin{cases} \frac{1}{\cos(SZA)} & \text{for } SZA \leq 35^\circ \\ \sqrt{TempB} * TempD & \text{for } 35^\circ < SZA < 90^\circ \\ \sqrt{TempB} * (2 * \text{Exp}(TempA) - TempD) & \text{for } SZA \geq 90^\circ \end{cases}$$

where:

$\text{Exp} =$ The Inverse Natural Log Function.

Calculate the Variance in the Grazing Incidence Function (VGIF).

$$TempE = \sqrt{\sin(SZA)} * \cos(SZA)$$

$$TempF = GIF * \left(\frac{1}{Radius} + (\text{Cos}(SZA))^2 \right)$$

$$TempG = \begin{cases} 0.0 & \text{for } SZA \leq 35^\circ \\ \frac{TempF - TempE}{2} & \text{for } 35^\circ < SZA < 90^\circ \\ \frac{TempF + TempE}{2} & \text{for } SZA \geq 90^\circ \end{cases}$$

$$TempH = GIF * \text{Cos}(SZA) * \text{Sin}(SZA) * \left(\frac{1}{2} - Radius \right)$$

$$TempI = Radius * (\text{Sin}(SZA))^3$$

$$TempJ = \begin{cases} \frac{\text{Sin}(SZA)}{(\text{Cos}(SZA))^2} & \text{for } SZA \leq 35^\circ \\ TempH + TempI & \text{for } 35^\circ < SZA < 90^\circ \\ TempH - TempI & \text{for } SZA \geq 90^\circ \end{cases}$$

$$VGIF = VRadius * (TempG)^2 + VSZA * (TempJ)^2$$

3.1.2.3 End

3.2 Derivation Constants

Cterm Predetermined scalar applied to the argument of the GIF error-function power-series approximation.

<i>Cterm</i>	$3.275911E - 01$
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DEA Predetermined Delta-Change increment of the Altitudes corresponding to the Elements in the current Electron Density Profile:

<i>DEA</i>	$5.0E + 00$
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VEA Predetermined Variance associated with the Altitude of the Current Element in the Electron Density Profile:

<i>VEA</i>	$0.0E + 00$
------------	-------------

Erefe Predetermined Reference Characteristic Electron Energy for the Analytical Model of Peak Auroral Ionization Production Rate Height due to Electrons:

<i>Erefe</i>	$1.0E + 00$
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VErefe Predetermined Variance associated with the Reference Characteristic Electron Energy for the Analytical Model of Peak Auroral Ionization Production Rate Height due to Electrons:

<i>VErefe</i>	$0.0E + 00$
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Erefp Predetermined Reference Characteristic Proton Energy for the Analytical Model of Peak Auroral Ionization Production Rate Height due to Protons:

<i>Erefp</i>	$4.0E + 00$
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VErefp Predetermined Variance associated with the Reference Characteristic Proton Energy for the Analytical Model of Peak Auroral Ionization Production Rate Height due to Protons:

$$\boxed{VErefp} \quad \boxed{0.0E+00}$$

HN Predetermined Neutral Scale Height:

$$\boxed{HN} \quad \boxed{9.0E+00}$$

VHN Predetermined Variance associated with the Neutral Scale Height:

$$\boxed{VHN} \quad \boxed{1.0E+00}$$

HO Predetermined Height of the Peak Photoionization Production Rate at the Subsolar Point due to Solar Photons:

$$\boxed{HO} \quad \boxed{1.08E+02}$$

VHO Predetermined Variance associated with the Height of the Peak Photoionization Production Rate at the Subsolar Point due to Solar Photons:

$$\boxed{VHO} \quad \boxed{1.6E+01}$$

MRE Predetermined Mean Radius of the Earth:

$$\boxed{MRE} \quad \boxed{6.375E+03}$$

VMRE Predetermined Variance associated with the Mean Radius of the Earth:

$$\boxed{VMRE} \quad \boxed{1.0E+02}$$

PERC Predetermined Peak Effective Recombination Coefficient:

$$\boxed{PERC} \quad \boxed{4.2E-07}$$

VPERC Predetermined Variance associated with the Peak Effective Recombination Coefficient:

<i>VPERC</i>	$3.97E - 15$
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PERCA Predetermined Altitude of the Peak Effective Recombination Coefficient:

<i>PERCA</i>	$1.08E + 02$
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VPERCA Predetermined Variance associated with the Altitude of the Peak Effective Recombination Coefficient:

<i>VPERCA</i>	$1.17E + 02$
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Pi Predetermined mathematical value of Pi.

<i>Pi</i>	$3.14159265358979E + 00$
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PPPRsubsolar Predetermined Peak Photoionization Production Rate at the Subsolar Point for Solar Photons:

<i>PPPRsubsolar</i>	$4.0E + 03$
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VPPPRsubsolar Predetermined Variance associated with the Peak Photoionization Production Rate at the Subsolar Point for Solar Photons:

<i>VPPPRsubsolar</i>	$2.0E + 05$
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PREFe Predetermined Reference Auroral Ionization Production Rate for the Analytical Model of Peak Auroral Ionization Production Rate for Electrons:

<i>PREFe</i>	$2.57E + 03$
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VPREFe Predetermined Variance associated with the Reference Auroral Ionization Production Rate for the Analytical Model of Peak Auroral Ionization Production Rate for Electrons:

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$VPREF_e$	$1.49E + 05$
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PREFp Predetermined Reference Auroral Ionization Production Rate for the Analytical Model of Peak Auroral Ionization Production Rate for Protons:

$PREF_p$	$5.4E + 03$
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VPREFp Predetermined Variance associated with the Reference Auroral Ionization Production Rate for the Analytical Model of Peak Auroral Ionization Production Rate for Protons:

$VPREF_p$	$2.62E + 06$
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SHPCe Predetermined Proportionality Constant for the Calculation of the Scale Height of the Auroral Ionization Production Rate for Electrons:

$SHPC_e$	$1.427E + 10$
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VSHPCe Predetermined Variance associated with the Proportionality Constant for the Calculation of the Scale Height of the Auroral Ionization Production Rate for Electrons:

$VSHPC_e$	$2.036329E + 18$
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SHPCp Predetermined Proportionality Constant for the Calculation of the Scale Height of the Auroral Ionization Production Rate for Protons:

$SHPC_p$	$2.3E + 10$
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VSHPCp Predetermined Variance associated with the Proportionality Constant for the Calculation of the Scale Height of the Auroral Ionization Production Rate for Protons:

$VSHPC_p$	$5.29E + 18$
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SHPF Predetermined Scale Height Proportionality Factor, independent of Electron or Proton Particle type (note that Exp is the Inverse Natural Log function):

$SHPF$	$\frac{1.0E - 05}{Exp(1.0)}$
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SHRC Predetermined Scale Height of the Effective Recombination Coefficient:

$SHRC$	$2.89E + 01$
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VSHRC Predetermined Variance associated with the Scale Height of the Effective Recombination Coefficient:

$VSHRC$	$8.3521E + 00$
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3.3 Derivation Coefficients

Cgif Predetermined Coefficients of the GIF Error-Function Power-Series Approximation.

	$C_{gif}(n)$
$n = 0$	$2.54829592E - 01$
$n = 1$	$-2.84496736E - 01$
$n = 2$	$1.421413741E + 00$
$n = 3$	$-1.453152027E + 00$
$n = 4$	$1.061405429E + 00$

CLBH1p Predetermined Analytical Model Coefficients of LBH1 (1400-1500A) yield versus Characteristic Energy for Protons:

	$CLBH1p(n)$
$n = 0$	$5.304845E + 00$
$n = 1$	$-4.509704E - 02$
$n = 2$	$1.935847E - 03$
$n = 3$	$-5.367008E - 05$

VLBH1p Predetermined Covariance Matrix of Analytical Model Coefficients of LBH1 (1400-1500A) yield versus Characteristic Energy for Protons:

$VLBH1p(i,j)$	$j = 0$	$j = 1$	$j = 2$	$j = 3$
$i = 0$	$3.358315E - 02$	$-1.357054E - 02$	$1.385237E - 03$	$-4.029744E - 05$
$i = 1$	$-1.357054E - 02$	$7.063619E - 03$	$-7.933817E - 04$	$2.427083E - 05$
$i = 2$	$1.385237E - 03$	$-7.933817E - 04$	$9.436205E - 05$	$-2.993938E - 06$
$i = 3$	$-4.029744E - 05$	$2.427083E - 05$	$-2.993938E - 06$	$9.747791E - 08$

CLBH2p Predetermined Analytical Model Coefficients of LBH2 (1650-1800A) yield versus Characteristic Energy for Protons:

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	$CLBH2p(n)$
$n = 0$	$4.791106E + 00$
$n = 1$	$3.945883E - 03$
$n = 2$	$-8.524076E - 06$
$n = 3$	$-1.700657E - 05$

VLBH2p Predetermined Covariance Matrix of Analytical Model Coefficients of LBH2 (1650-1800A) yield versus Characteristic Energy for Protons:

$VLBH2p(i,j)$	$j = 0$	$j = 1$	$j = 2$	$j = 3$
$i = 0$	$1.829806E - 02$	$-5.942663E - 03$	$5.369682E - 04$	$-1.444207E - 05$
$i = 1$	$-5.942663E - 03$	$2.346130E - 03$	$-2.319578E - 04$	$6.570856E - 06$
$i = 2$	$-2.319578E - 04$	$-7.933817E - 04$	$2.412197E - 05$	$-7.060817E - 07$
$i = 3$	$6.570856E - 06$	$2.427083E - 05$	$-7.060817E - 07$	$2.114684E - 08$

CEOEe Predetermined Analytical Model Coefficients of Characteristic Energy versus LBH Ratio for Electrons:

	$CEOEe(n)$
$n = 0$	$-1.059909E + 00$
$n = 1$	$3.260567E + 00$

VEOEe Predetermined Covariance Matrix of Analytical Model Coefficients of Characteristic Energy versus LBH Ratio for Electrons:

$VEOEe(i,j)$	$j = 0$	$j = 1$
$i = 0$	$3.068972E - 02$	$5.806447E - 03$
$i = 1$	$5.806447E - 03$	$1.593687E - 05$

CLBH1e Predetermined Analytical Model Coefficients of LBH1 (1400-1500A) yield versus Characteristic Energy for Electrons:

	$CLBH1e(n)$
$n = 0$	$5.427762E + 00$
$n = 1$	$-4.822704E - 01$
$n = 2$	$2.751889E - 02$
$n = 3$	$-6.613106E - 04$

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VLBH1e Predetermined Covariance Matrix of Analytical Model Coefficients of LBH1 (1400-1500A) yield versus Characteristic Energy for Electrons:

$VLBH1e(i,j)$	$j = 0$	$j = 1$	$j = 2$	$j = 3$
$i = 0$	$2.622690E - 02$	$-1.666057E - 02$	$2.287335E - 03$	$-8.501208E - 05$
$i = 1$	$-1.666057E - 02$	$1.532884E - 02$	$-2.314953E - 03$	$8.985284E - 05$
$i = 2$	$2.287335E - 03$	$-2.314953E - 03$	$3.690269E - 04$	$-1.480610E - 05$
$i = 3$	$-8.501208E - 05$	$8.985284E - 05$	$-1.480610E - 05$	$6.082223E - 07$

CLBH2e Predetermined Analytical Model Coefficients of LBH2 (1650-1800A) yield versus Characteristic Energy for Electrons:

	$CLBH2e(n)$
$n = 0$	$4.528492E + 00$
$n = 1$	$-4.950725E - 02$
$n = 2$	$-2.789456E - 03$
$n = 3$	$1.668927E - 04$

VLBH2e Predetermined Covariance Matrix of Analytical Model Coefficients of LBH2 (1650-1800A) yield versus Characteristic Energy for Electrons:

$VLBH2e(i,j)$	$j = 0$	$j = 1$	$j = 2$	$j = 3$
$i = 0$	$8.797960E - 03$	$-4.462779E - 03$	$5.769323E - 04$	$-2.096335E - 05$
$i = 1$	$-4.462779E - 03$	$2.704346E - 03$	$-3.780627E - 04$	$1.432688E - 05$
$i = 2$	$5.769323E - 04$	$-3.780627E - 04$	$5.595920E - 05$	$-2.198212E - 06$
$i = 3$	$-2.096335E - 05$	$1.432688E - 05$	$-2.198212E - 06$	$8.861385E - 08$

CEOPp Predetermined Analytical Model Coefficients of Characteristic Energy versus LBH Ratio for Protons:

	$CEOPp(n)$
$n = 0$	$-3.251152E + 01$
$n = 1$	$5.375951E + 01$

VEOPp Predetermined Covariance Matrix of Analytical Model Coefficients of Characteristic Energy versus LBH Ratio for Protons:

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$VEOPp(i,j)$	$j = 0$	$j = 1$
$i = 0$	$2.463820E + 02$	$-3.120715E + 02$
$i = 1$	$-3.120715E + 02$	$4.048413E + 02$

CLYAp Predetermined Analytical Model Coefficients of Lyman Alpha (1216A) yield versus Characteristic Energy for Protons:

	$CLYAp(n)$
$n = 0$	$9.969755E + 00$
$n = 1$	$-2.896852E - 01$
$n = 2$	$1.729508E - 02$
$n = 3$	$-3.962961E - 04$

VLyAp Predetermined Covariance Matrix of Analytical Model Coefficients of Lyman Alpha (1216A) yield versus Characteristic Energy for Protons:

$VLyAp(i,j)$	$j = 0$	$j = 1$	$j = 2$	$j = 3$
$i = 0$	$2.087800E - 04$	$-7.852403E - 05$	$7.609171E - 06$	$-2.135304E - 07$
$i = 1$	$-7.852403E - 05$	$3.974046E - 05$	$-4.283232E - 06$	$1.270805E - 07$
$i = 2$	$7.609171E - 06$	$-4.283232E - 06$	$4.866846E - 07$	$-1.493402E - 08$
$i = 3$	$-2.135304E - 07$	$1.270805E - 07$	$-1.493402E - 08$	$4.692712E - 10$

CHMAXe Predetermined Analytical Model Coefficients of Peak Auroral Ionization Production Rate Height versus Characteristic Energy for Electrons:

	$CHMAXe(n)$
$n = 0$	$2.079230E + 00$
$n = 1$	$-9.412050E - 02$

VHMAXe Predetermined Covariance Matrix of Analytical Model Coefficients of Peak Auroral Ionization Production Rate Height versus Characteristic Energy for Electrons:

$VHMAXe(i,j)$	$j = 0$	$j = 1$
$i = 0$	$1.703090E - 03$	$-2.176790E - 03$
$i = 1$	$-2.176790E - 03$	$3.265280E - 03$

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CHMAXp Predetermined Analytical Model Coefficients of Peak Auroral Ionization Production Rate Height versus Characteristic Energy for Protons:

	$CHMAXp(n)$
$n = 0$	$2.078000E + 00$
$n = 1$	$-4.072000E - 02$

VHMAXp Predetermined Covariance Matrix of Analytical Model Coefficients of Peak Auroral Ionization Production Rate Height versus Characteristic Energy for Protons:

$VHMAXp(i, j)$	$j = 0$	$j = 1$
$i = 0$	$4.000000E - 02$	$2.000000E - 03$
$i = 1$	$2.000000E - 03$	$6.000000E - 06$

CPMAXe Predetermined Analytical Model Coefficients of Peak Auroral Ionization Production Rate versus Characteristic Energy for Electrons:

	$CPMAXe(n)$
$n = 0$	$0.000000E + 00$
$n = 1$	$9.257770E - 01$
$n = 2$	$-5.032010E - 01$

VPMAXe Predetermined Covariance Matrix of Analytical Model Coefficients of Peak Auroral Ionization Production Rate versus Characteristic Energy for Electrons:

$VPMAXe(i, j)$	$j = 0$	$j = 1$	$j = 2$
$i = 0$	$0.000000E + 00$	$0.000000E + 00$	$0.000000E + 00$
$i = 1$	$0.000000E + 00$	$1.347350E - 01$	$-1.639950E - 01$
$i = 2$	$0.000000E + 00$	$-1.639950E - 01$	$2.142490E - 01$

CPMAXp Predetermined Analytical Model Coefficients of Peak Auroral Ionization Production Rate versus Characteristic Energy for Protons:

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	$CPMAXp(n)$
$n = 0$	$0.000000E + 00$
$n = 1$	$3.507660E - 01$
$n = 2$	$-8.847370E - 02$

VPMAXp Predetermined Covariance Matrix of Analytical Model Coefficients of Peak Auroral Ionization Production Rate versus Characteristic Energy for Protons:

$VPMAXp(i,j)$	$j = 0$	$j = 1$	$j = 2$
$i = 0$	$0.000000E + 00$	$0.000000E + 00$	$0.000000E + 00$
$i = 1$	$0.000000E + 00$	$3.229050E - 01$	$-5.360260E - 01$
$i = 2$	$0.000000E + 00$	$-5.360260E - 01$	$9.333660E - 01$

3.4 Figures

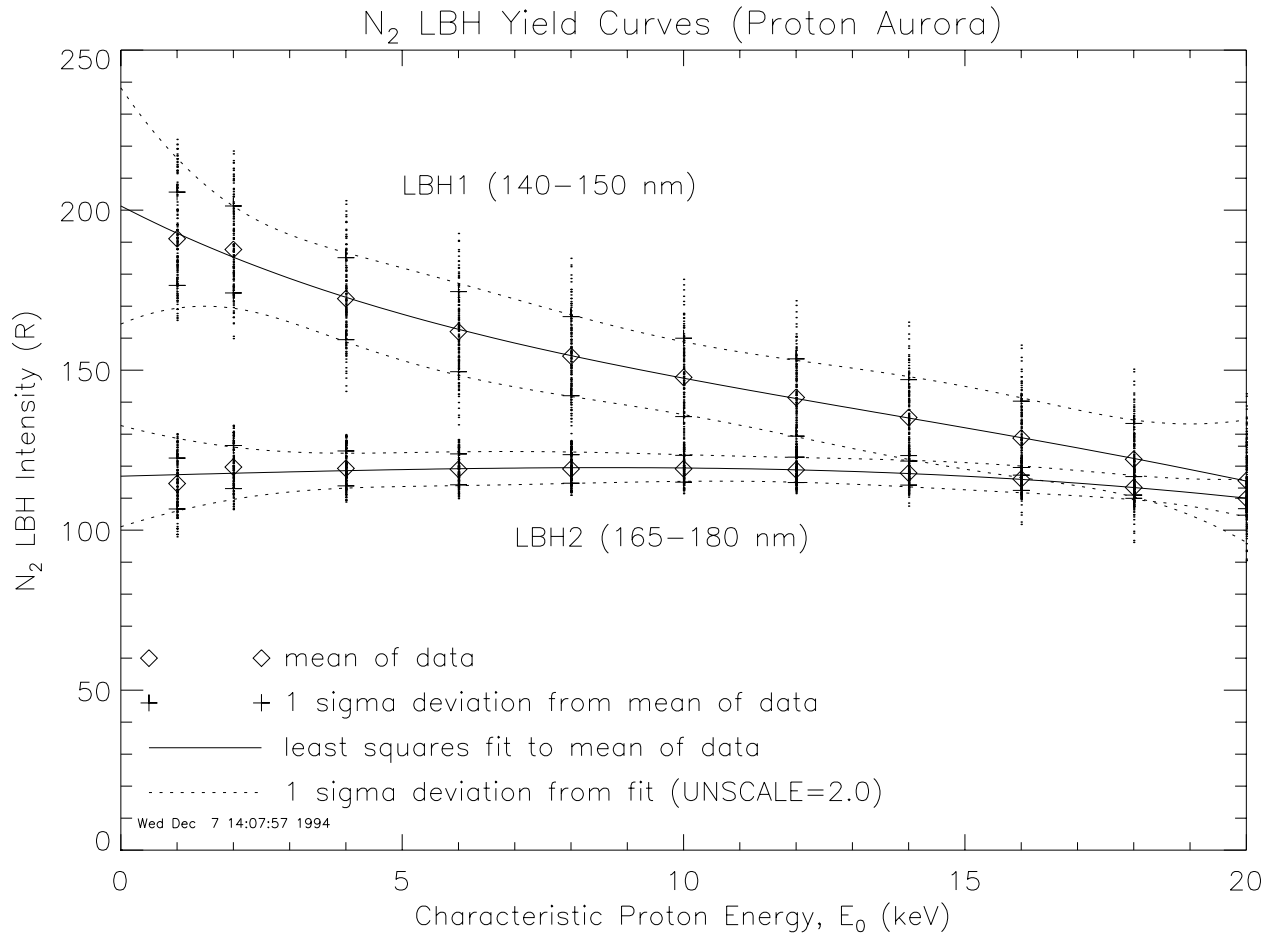


Figure 1 N₂ LBH1 (140-150 nm) and LBH2 (156-180 nm) yield curves for pure proton aurora. The dots represent model values for a wide range of geophysical conditions. At each energy, the sample mean and sample standard deviation were calculated. The mean values were fit using a weighted least squares method in which the weights were the reciprocal of twice the sample standard deviation. The 1σ deviation from the least squares fit was obtained using the covariance matrix produced by the least squares fitting algorithm.

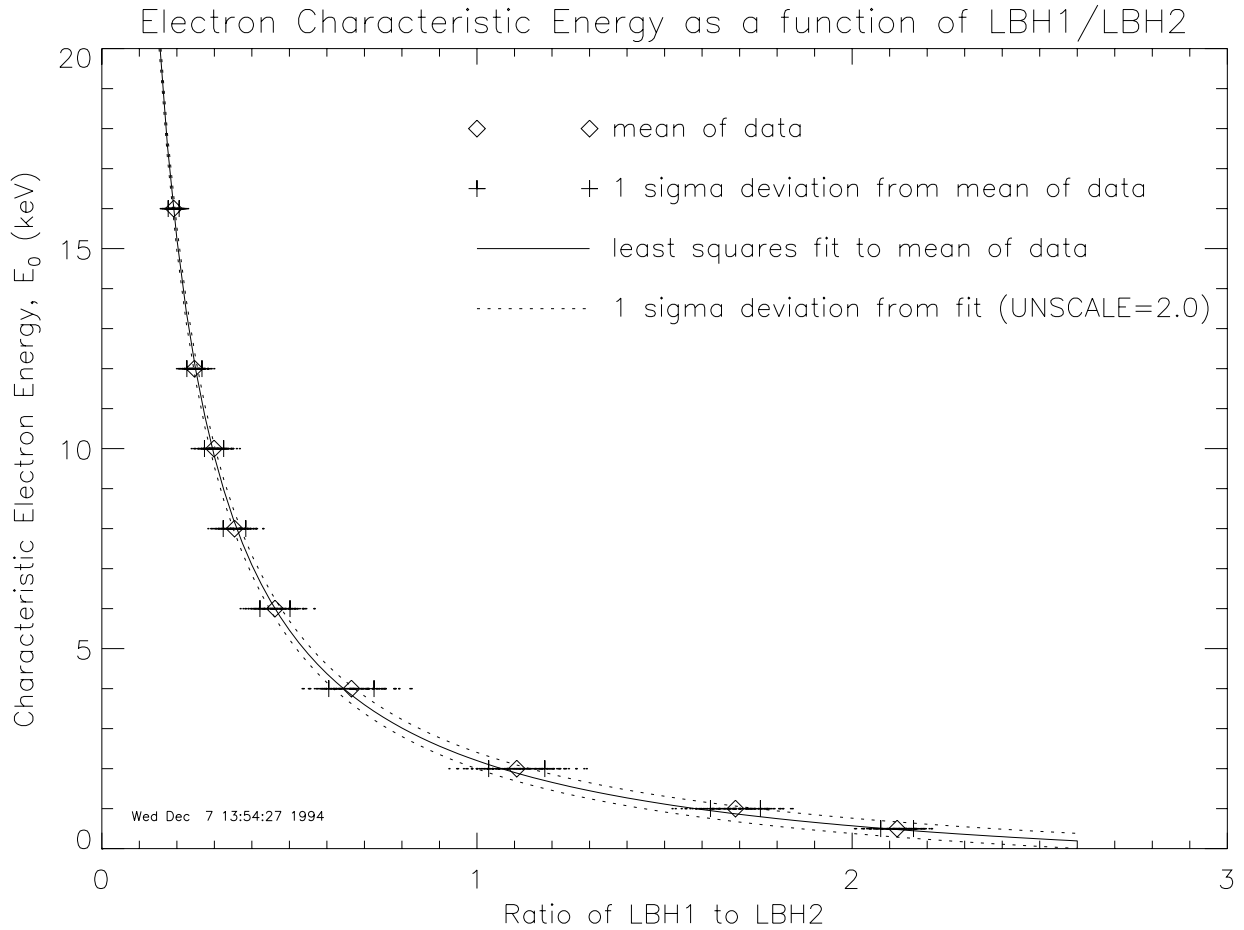


Figure 2 Electron characteristic energy $E_{0,e}$ as a function of the LBH1/LBH2 ratio. In this case, the inverse ratio was first fit as a linear function of $E_{0,e}$, then inverted to produce $E_{0,e}$ as a linear function of the inverse ratio. The covariance matrix of the second form was derived from the covariance matrix of the first form as described in the text. As in Figure 1, the weights were the reciprocal of twice the sample standard deviation.

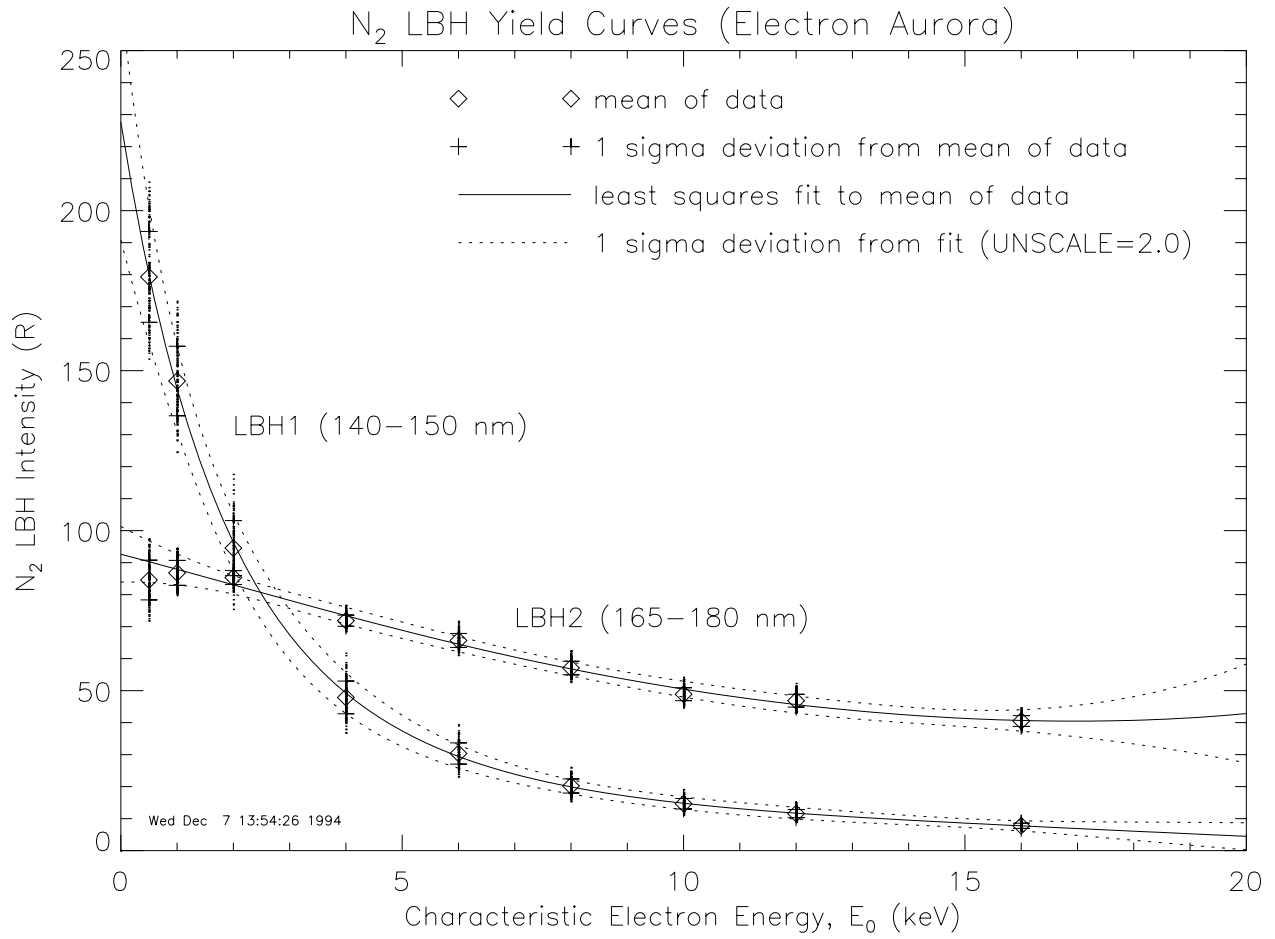


Figure 3 N₂ LBH1 (140-150 nm) and LBH2 (165-180 nm) yield curves for pure electron aurora. The fitting process was the same as for Figure 1.

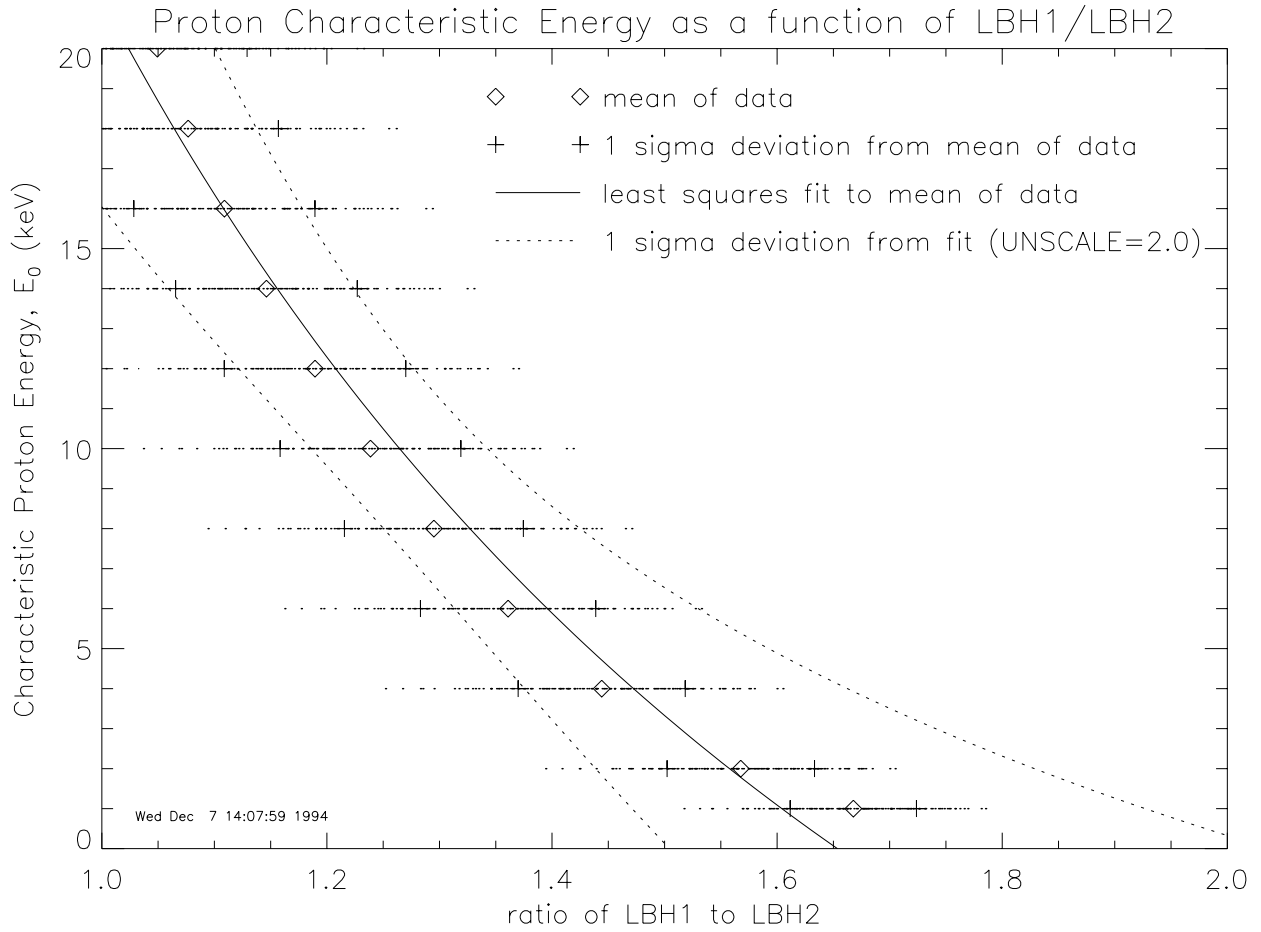


Figure 4 Proton characteristic energy $E_{0,p}$ as a function of the LBH1/LBH2 ratio. The fitting process was the same as for Figure 2.

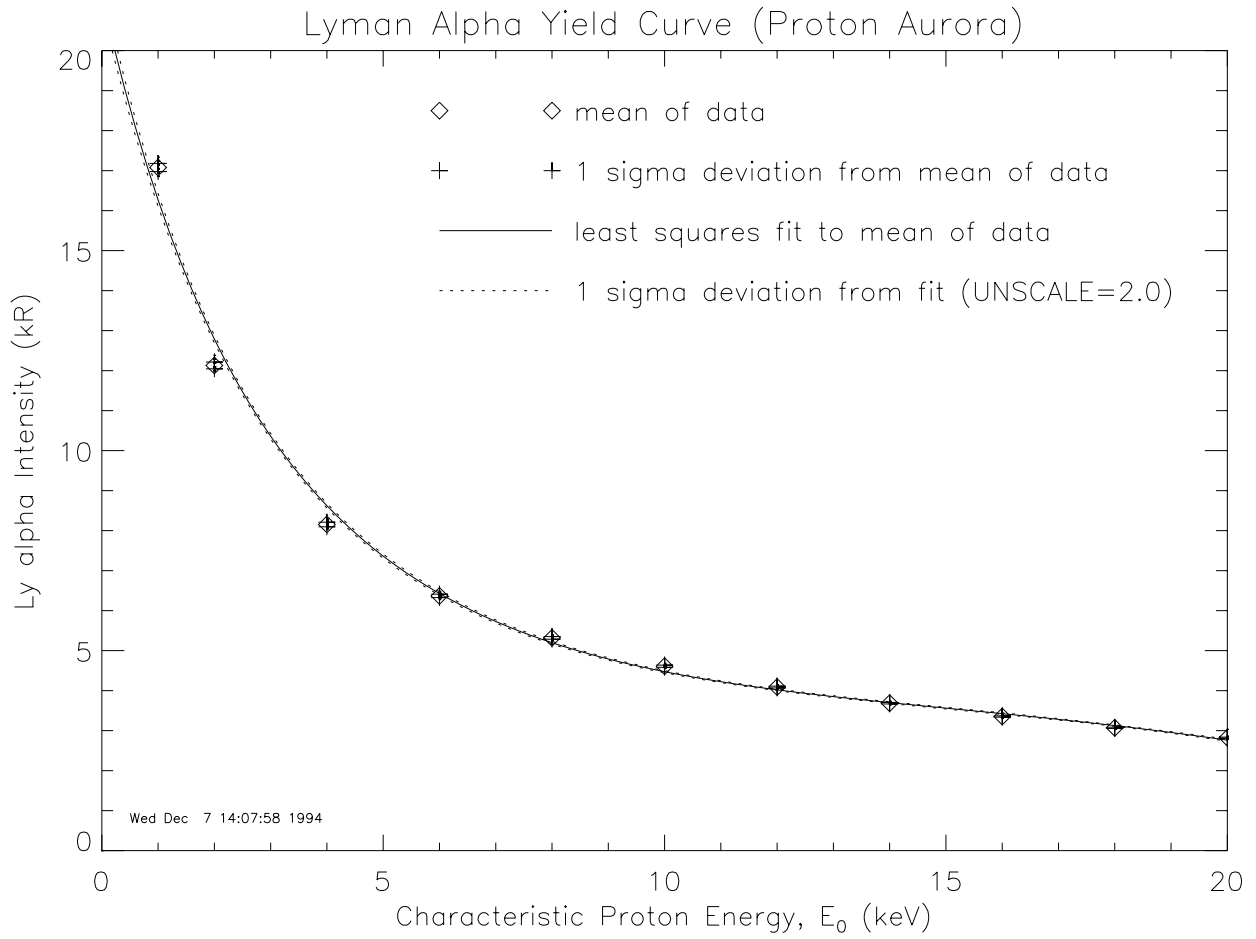


Figure 5 Hydrogen Lyman alpha yield curve for pure proton aurora. The fitting process was the same as described in Figure 1.

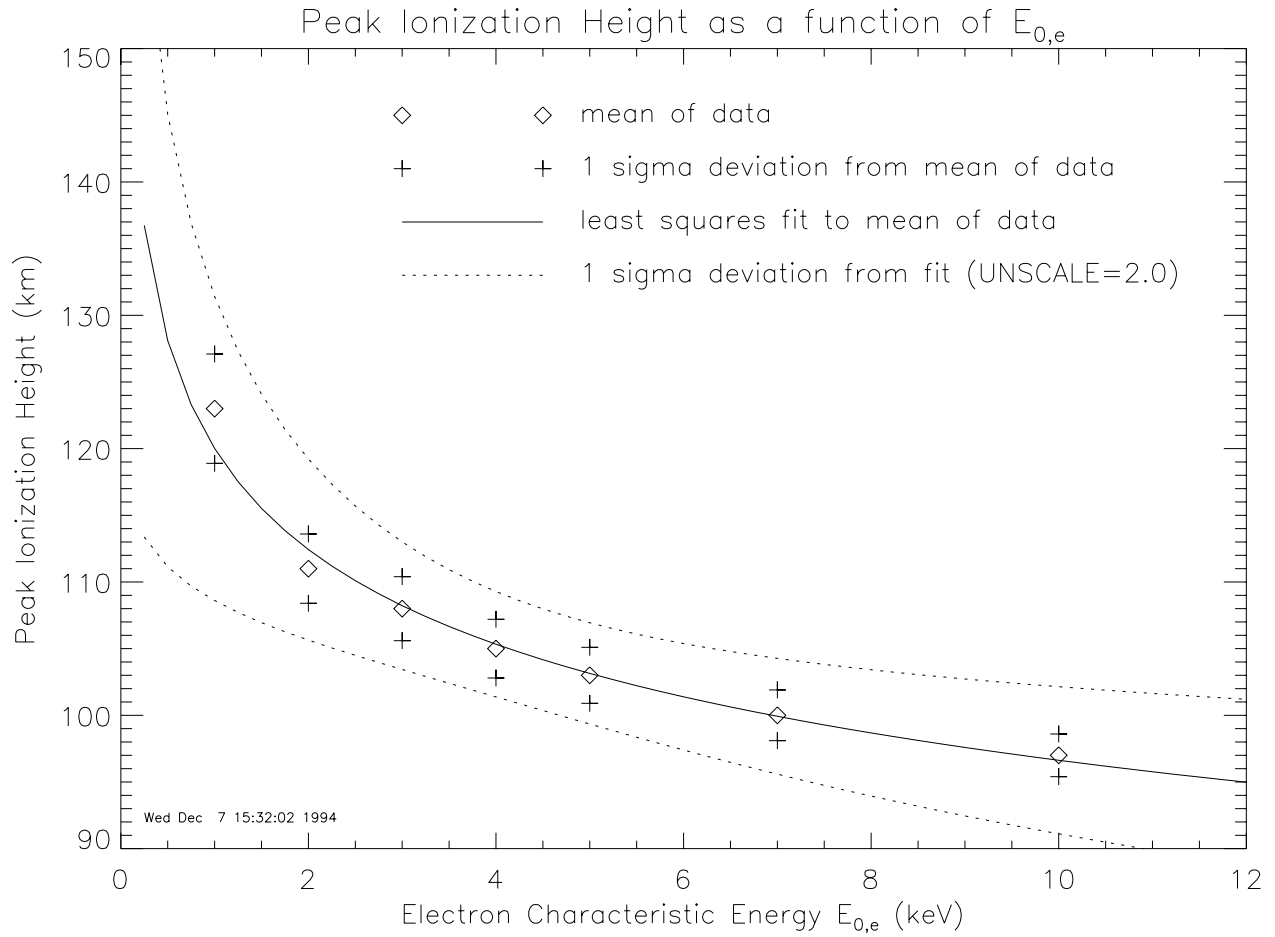


Figure 6 Peak ionization height as a function of electron characteristic energy. The fitting process was the same as described in Figure 1.

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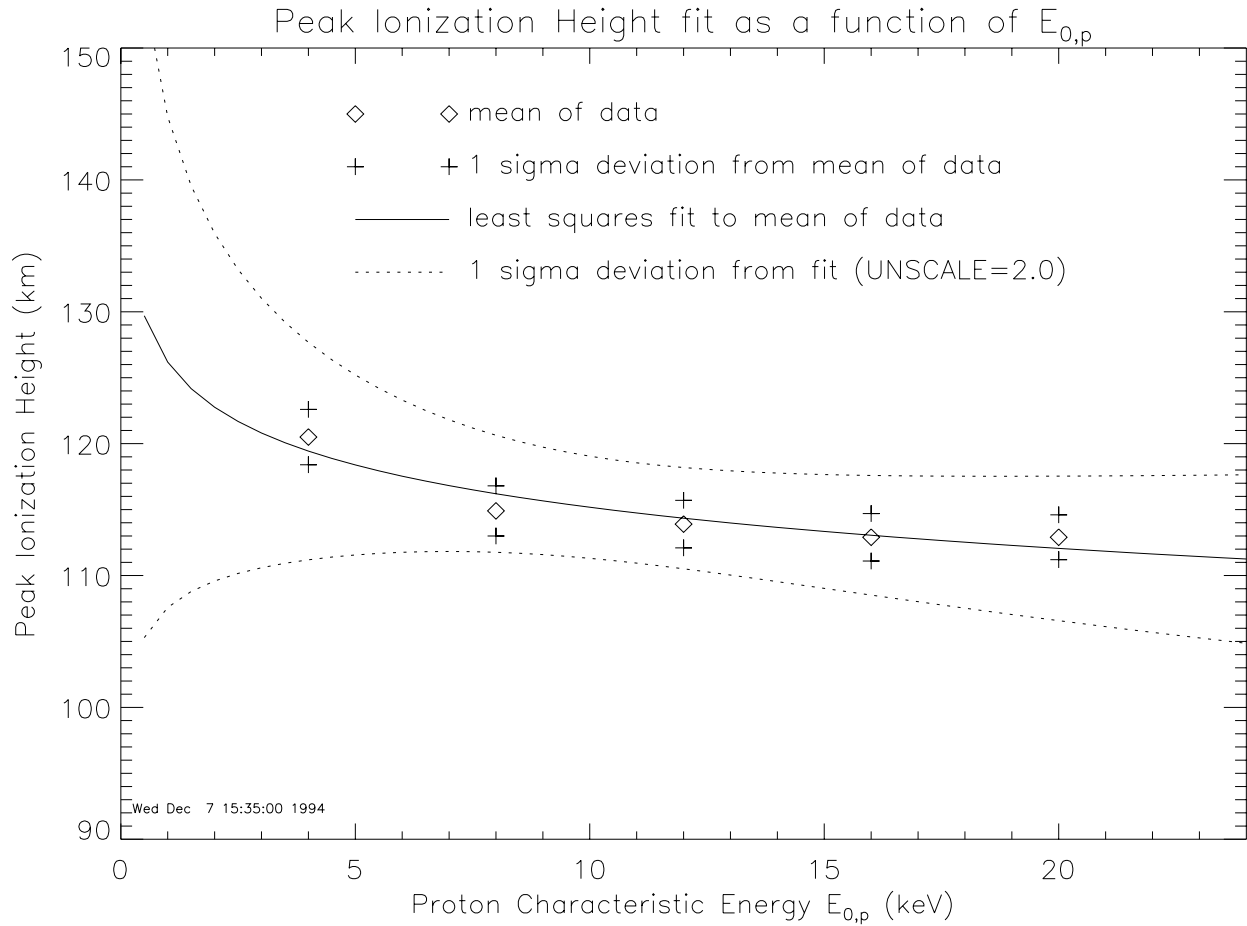


Figure 7 Peak ionization height as a function of proton characteristic energy. The fitting process was the same as described in Figure 1.

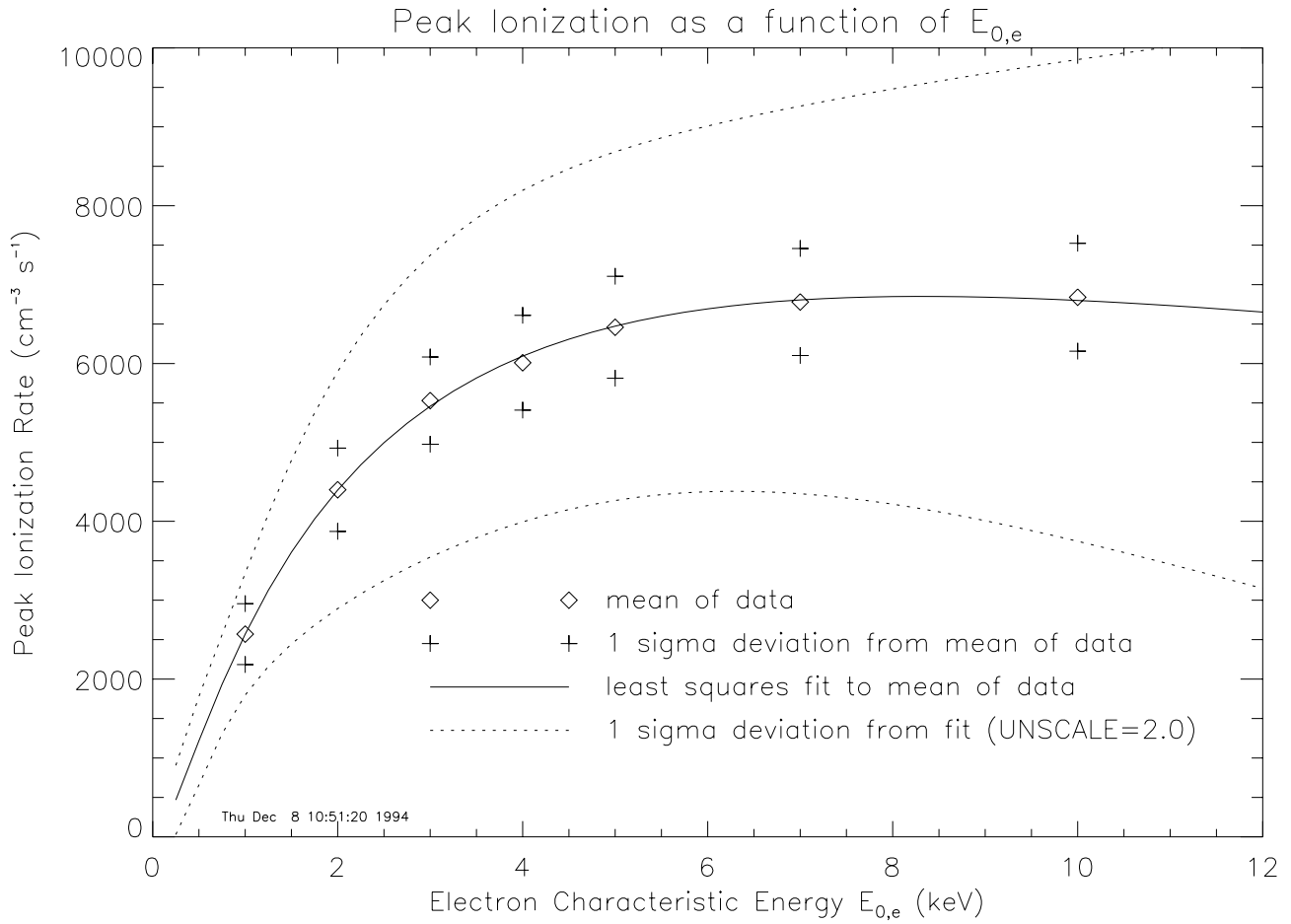


Figure 8 Peak ionization rate as a function of electron characteristic energy. The fitting process was the same as described in Figure 1.

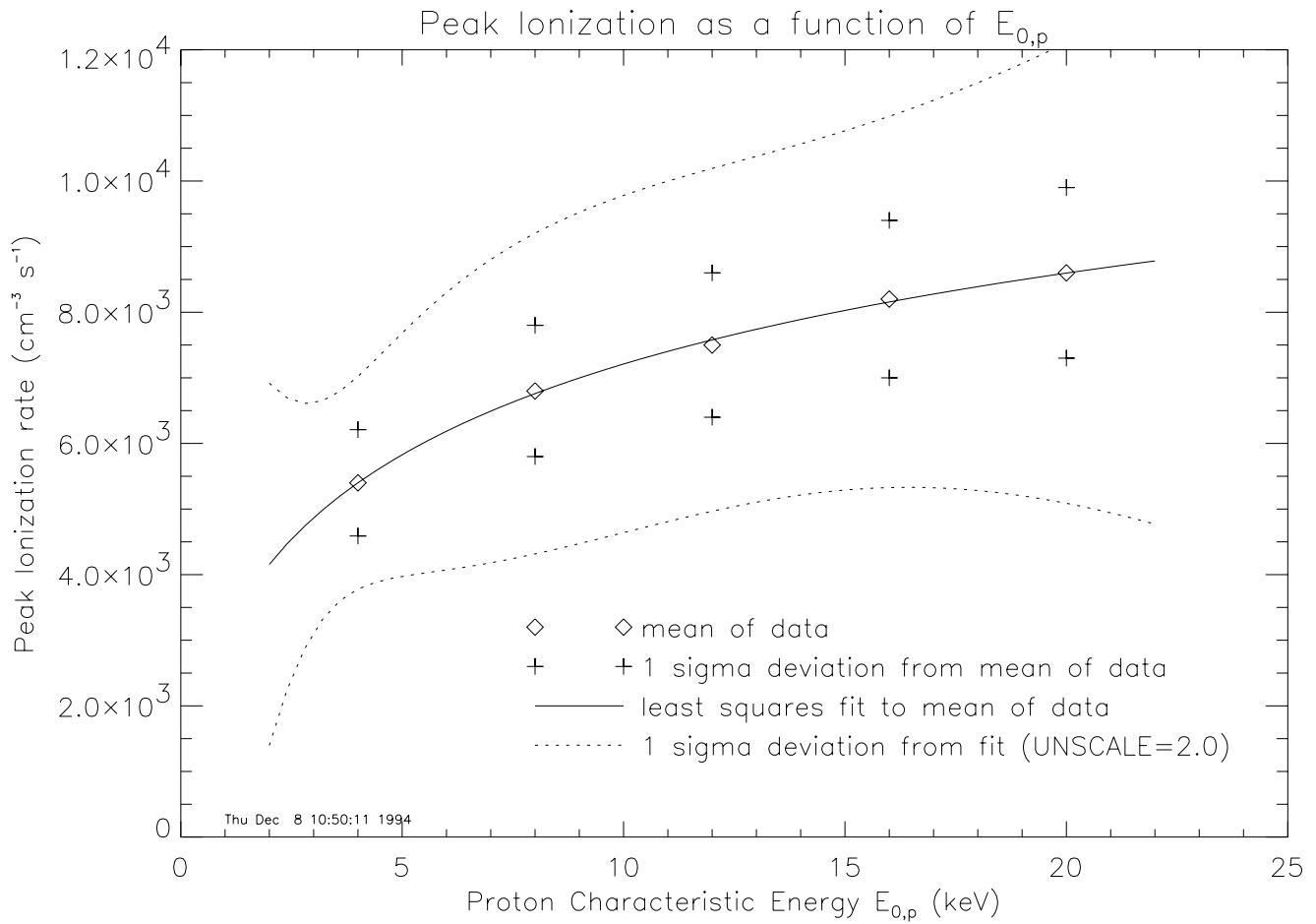


Figure 9 Peak ionization rate as a function of proton characteristic energy. The fitting process was the same as described in Figure 1.

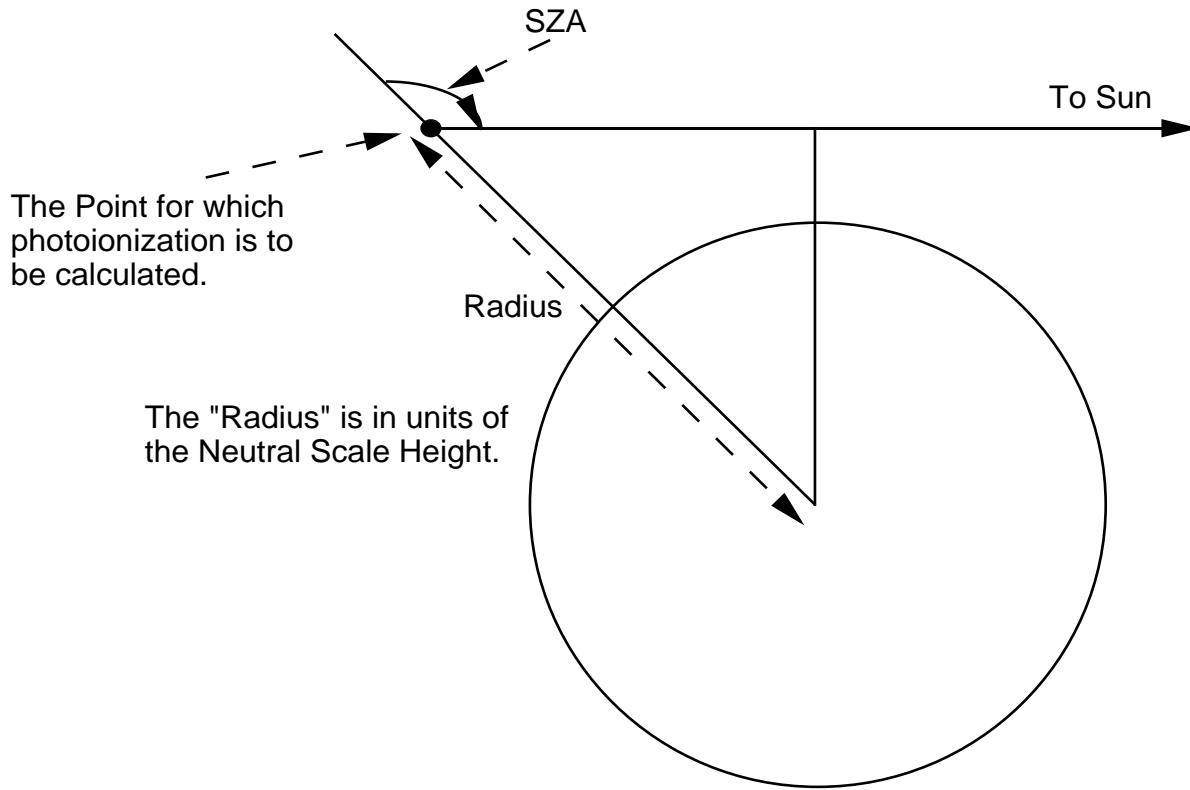


Figure 10 Geometry of the Chapman Grazing Incidence Function and the Photoionization Calculation.

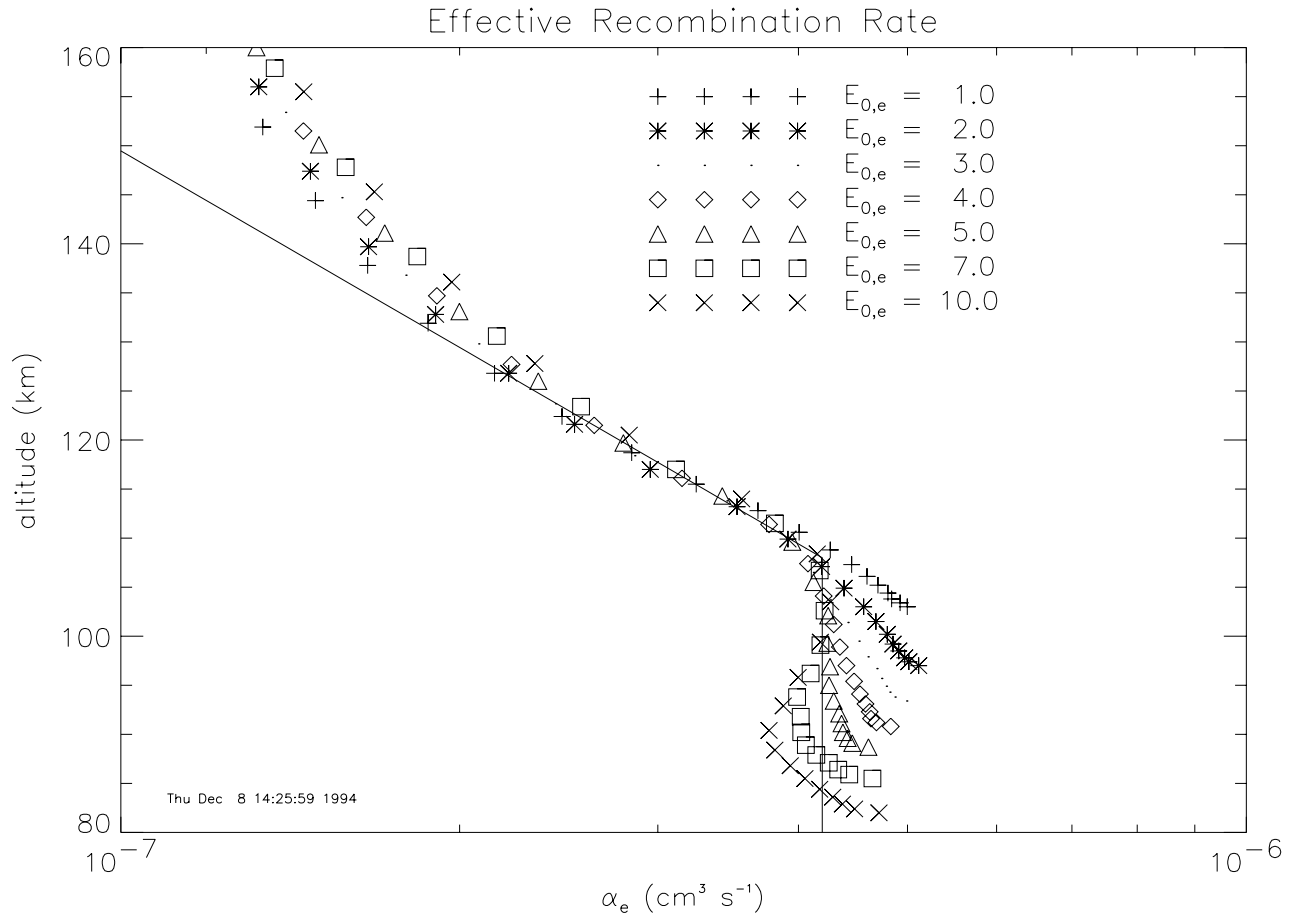


Figure 11 The effective recombination coefficient as a function of altitude for a variety of incident particle spectra. The solid line is the approximation used in this algorithm. See text for more explanation.

3.5 References

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4. Document Modification History

This section describes **all modifications** made to the SSUSI Auroral E-Region Algorithm **since the Version 1.1 release dated 22-Jul-1994**.

Author (Name, Org, Date)	Modification
R. Weed, JHU-APL, 30Dec1994	Added Section 4. (Document Modification History).
R. Weed, JHU-APL, 05Jan1995	Implemented all PL/CPI modifications, as mandated at SSUSI CDR. Of note was the deletion of section 2.7 (seperate re-derivation of NmE), the addition of textual descriptions and reasoning throughout, and the addition of a series of figures (see Appendix) showing typical data plots which lead to the representative equations.
R. Weed, JHU-APL, 05Jan1995	Added Critical Frequency (Plasma Frequency FoE) derivation section 2.7.
R. Weed, JHU-APL, 05Jan1995	Renumbered derivation steps in section 2.4 (E0p) to eliminate gaps left by the PL/CPI modifications. Renumbered sections 2.4.2.8 through 2.4.2.12 to be 2.4.2.2 through 2.4.2.6.
R. Weed, JHU-APL, 05Jan1995	Modified text and step descriptions in section 2.6 to reflect the derivation of both HmE and NmE.
R. Weed, JHU-APL, 05Jan1995	Added Covariance term as input into the E0p derivation to support the variance calculation VR12P.
R. Weed, JHU-APL, 06Jan1995	Removed the "brief" description of the SSUSI instrument from section 1.1.
R. Weed, JHU-APL, 07Jan1995	Changed array notation from () to []. Enclosed in () every data item that is raised to a power.

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- R. Weed, JHU-APL, 07Jan1995 Changed notation used for the Natural Log and the Inverse Natural Log ("ln(x)" is now "Log_e(x)", and "e^x" is now "Exp(x)"). All intrinsic functions are now in a common, separate font.
- R. Weed, JHU-APL, 08Jan1995 Added Flow-Chart to section 2.1 to detail the overall derivation processing.
- R. Weed, JHU-APL, 09Jan1995 Reordered derivations as they appear in the LID from:
E0e, Qe, E0p, Qp, HmE, NmE, FoE
to:
Qp, E0e, Qe, E0p, HmE, NmE, FoE
along with a corresponding change in the section numbers. The order of appearance in the LID now better reflects the required sequence of derivations.
- R. Weed, JHU-APL, 09Jan1995 Incremented the Document Version Number from 1.1 to 2.0.